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On subcategories closed under predecessors and the representation dimension

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A B S T R A C T

Let Λ be an artin algebra, $\mathcal{A}$ and $\mathcal{C}$ be full subcategories of the category of finitely generated Λ-modules consisting of indecomposable modules and closed under predecessors and successors respectively. In this paper we relate, under various hypotheses, the representation dimension of Λ to those of the left support algebra of $\mathcal{A}$ and the right support algebra of $\mathcal{C}$. Our results are then applied to the classes of laura algebras, ada algebras and Nakayama oriented pullbacks.

Introduction

The aim of the representation theory of artin algebras is to characterize and to classify algebras using properties of module categories. The representation dimension of an artin algebra was introduced by Auslander [9] and he expected that this invariant would give a measure of how far an algebra is from being representation-finite. He proved that a non-semisimple algebra Λ is representation-finite if and only if its representation

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dimension rep.dim Λ is two. Iyama proved that the representation dimension of an artin algebra is always finite (see [19]) and Rouquier has constructed examples of algebras with rep.dim $\Lambda = r$ for any $r \geq 2$ (see [23]).

Igusa and Todorov gave in [18] an interesting connection with the finitistic dimension conjecture. They proved that if Λ has representation dimension at most three then its finitistic dimension is finite.

Auslander proved in [9] that if Λ is a hereditary algebra, then rep.dim Λ is at most three. Many other classes of algebras have representation dimension at most three, as for example, tilted and laura algebras [7], trivial extensions of hereditary algebras [14] and quasi-tilted algebras [22]. Other results can be found also in [15,26].

In order to calculate the representation dimension of an artin algebra Λ, one reasonable approach would be to split the module category mod Λ of the finitely generated modules into pieces and calculate the representation dimension of algebras associated to each piece. In this sense, we consider for a full subcategory $\mathcal{C}$ of ind Λ closed under successors its support algebra $\Lambda_{\mathcal{C}}$, in the sense of [2], and for a full subcategory $\mathcal{A}$ of ind Λ closed under predecessors its support algebra ${}_{\mathcal{A}}\Lambda$. Our two main theorems (Theorems 2.6 and 4.2) relate rep.dim Λ to rep.dim ${}_{\mathcal{A}}\Lambda$ or rep.dim $\Lambda_{\mathcal{C}}$ when $\mathcal{A}$ and $\mathcal{C}$ satisfy some additional hypotheses.

Before stating our first main theorem, we need to recall some definitions. Let Λ be an artin algebra and ind Λ be a full subcategory of mod Λ consisting of one representative from each isomorphism class of indecomposable modules. A trisection of ind Λ is a triple of disjoint full subcategories $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ such that ind $\Lambda = \mathcal{A} \cup \mathcal{B} \cup \mathcal{C}$ and $\mathrm{Hom}(\mathcal{C}, \mathcal{B}) = \mathrm{Hom}(\mathcal{C}, \mathcal{A}) = \mathrm{Hom}(\mathcal{B}, \mathcal{A}) = 0$, see [1]. We say that $\mathcal{B}$ is finite if it contains only finitely many objects of ind Λ. We denote by $\mathcal{L}_\Lambda$ and $\mathcal{R}_\Lambda$, respectively, the left and the right parts of mod Λ in the sense of [16] (or see Section 1.2 below). For the definition of covariantly and contravariantly finite subcategories, we refer the reader to [12] (or see Section 1.3 below).

The first theorem is the following:

Theorem. *Let Λ be a representation-infinite artin algebra and $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ be a trisection of* ind Λ *with $\mathcal{B}$ finite.*

(a) *If $\mathcal{C} \subseteq \mathcal{R}_\Lambda$ and* add $\mathcal{C}$ *is covariantly finite, then*

$$\text{rep.dim}\,\Lambda = \max\{3, \text{rep.dim}\,{}_{\mathcal{A}}\Lambda\}.$$

(b) *If $\mathcal{A} \subseteq \mathcal{L}_\Lambda$ and* add $\mathcal{A}$ *is contravariantly finite, then*

$$\text{rep.dim}\,\Lambda = \max\{3, \text{rep.dim}\,\Lambda_{\mathcal{C}}\}.$$

As consequences of this theorem we prove that the class of ada algebras, introduced and studied in [3], has representation dimension at most three (Corollary 5.3), and give