Journal of Algebra 417 (2014) 148–160

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

A fast algorithm for constructing Arf closure and a conjecture

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ARTICLE INFO

Article history:
Received 18 September 2013
Available online 19 July 2014
Communicated by Seth Sullivant

Keywords:
Branch
Arf ring
Arf closure
Arf semigroup
Hilbert function

ABSTRACT

In this article, we give a fast and an easily implementable algorithm for computing the Arf closure of an irreducible algebroid curve (or a branch). Moreover, we study the relation between the branches having the same Arf closure and their regularity indices. We give some results and a conjecture, which are steps towards the interpretation of Arf closure as a specific way of taming the singularity.
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1. Introduction

Canonical closure of a local ring constructed by Cahit Arf solves the problem of determining the characters of a space curve singularity [1]. The characters of a plane curve singularity, introduced first by Du Val in 1942, are special integers, which determine the multiplicity sequence of the plane curve singularity, [15]. Contrary to the well-known plane case, in which the characteristic exponents, the multiplicity sequence, the semigroup of the singularity and the characters determine each other, it was not known how

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http://dx.doi.org/10.1016/j.jalgebra.2014.06.023
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to obtain the characters in the space case, until Cahit Arf developed his theory [1]. In 1946, Arf showed that the characters of a space branch could be obtained from the completion of the local ring corresponding to the branch by constructing its canonical closure, later known as Arf closure [1]. Since then, many algebraic geometers and algebraists have worked on Arf rings and Arf closure, [3,8,9,13,19]. Moreover, Arf semigroups and their applications in coding theory have been a recent area of interest [5,6,10,14,20]. For a good survey and a quick introduction to Arf theory, see [22].

In spite of all this interest in Arf rings and Arf closure, there is not a fast implementable algorithm for the computation of Arf closure in the literature. The construction method given by Arf cannot be implemented as an algorithm, without finding a bound, that determines up to which degree a division series should be expanded, and finding an efficient bound is not easy at all. The construction of Arf closure by using Hamburger–Noether matrices presented by Castellanos does not give an answer to this problem either [12]. The only implemented algorithm is given by Arslan [2]. The algorithm uses Arf's construction method and starts with determining the semigroup of values of the branch and its conductor, but determining the semigroup of values and the conductor of a branch is a difficult problem, which has been studied by many mathematicians and different algorithms have been given [11,18]. Noting that the special case of this problem is the famous Fobenius problem (or coin problem) makes it clear, why this problem is difficult, and it is unnecessary to mention that there is a vast literature on the Frobenius problem.

Our main objects of interest in this article are space curve singularities. Following Castellanos and Castellanos [12] and using their notation, we consider a space curve singularity as an algebroid curve $C = \operatorname{Spec}(R)$, where $(R, \mathfrak{m}, k)$ is a local ring, complete for the $\mathfrak{m}$-adic topology, with Krull dimension 1, and having k as a coefficient field. We work with irreducible algebroid curves (or branches), in other words R will always be a domain. In this case, it can be shown that $R \cong k[\![\varphi_1(t), ..., \varphi_n(t)]\!] \subset k[\![t]\!]$, where $\varphi_1(t), ..., \varphi_n(t)$ are power series in t, see [12] or [18]. Hence, we will be working with subrings of $k[\![t]\!]$. Here, the set $\{\varphi_1(t), ..., \varphi_n(t)\}$ is a parametrization of the curve C and the minimum possible n is called its embedding dimension. It is denoted by $\operatorname{embdim}(C)$ and is also equal to $\dim_k \mathfrak{m}/\mathfrak{m}^2$. The minimum order of the series of any parametrization of the curve C is called its multiplicity, which is also equal to the multiplicity of the local ring R.

By using the notation in [22], we denote the semigroup of orders of the local ring $R = k[\![\varphi_1(t), ..., \varphi_n(t)]\!]$ by $W(R)$. For $n \in \mathbb{N}$, $I_n = \{r \in R \mid \operatorname{ord}(r) \geq n\}$ and $I_n/S_n = \{r \cdot S_n^{-1} \mid \operatorname{ord}(r) \geq n\}$, where $S_n \in R$ has order n. In general, I_n/S_n is not a ring. The ring generated by I_n/S_n is denoted by $[I_n]$. A ring is called an Arf ring, if $I_n/S_n = [I_n]$ for any n in its semigroup of orders. For a local ring $R \subset k[\![t]\!]$, the smallest Arf ring containing R is called the Arf closure of R. In [1], Arf not only defines the Arf closure, but also gives a method for its construction: The Arf closure of $R = k[\![\varphi_1(t), ..., \varphi_n(t)]\!]$ can be presented as $R^* = k + kF_0 + kF_0F_1 + ... + kF_0...F_{l-2} + k[\![t]\!]F_0...F_{l-1}$, where $R_0 = R$, F_i is a smallest ordered element of R_i with $a_i = \operatorname{ord}(F_i)$ and $R_i = [I_{a_{i-1}}]$ for $i = 1, ..., l$ and $R_l = k[\![t]\!]$. Hence, that $W(R^*) = \{0, a_0, a_0 + a_1, ..., a_0 + ... + a_{l-2}, a_0 + ... + a_{l-1} + \mathbb{N}\}$ and

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