Journal of Algebra 416 (2014) 1–24

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

A characterization of cycle-finite generalized double tilted algebras

Adam Skowyrski

Faculty of Mathematics and Computer Science, Nicolaus Copernicus University, Chopina 12/18, 87-100 Toruń, Poland

ARTICLE INFO

Article history:
Received 16 December 2013
Available online 2 July 2014
Communicated by Changchang Xi

MSC:
16G10
16G60
16G70

Keywords:
Tilted algebra
Generalized double tilted algebra
Cycle-finite algebra
Short chain
Auslander–Reiten quiver

ABSTRACT

We provide a new characterization of cycle-finite generalized double tilted algebras by the existence of a faithful module which is the middle of at most finitely many short chains.

© 2014 Elsevier Inc. All rights reserved.

1. Introduction and the main results

Throughout the paper, by an algebra we mean a basic indecomposable artin algebra over a commutative artin ring K. For an algebra A, we denote by $\operatorname{mod} A$ the category of finitely generated right A-modules, by $\operatorname{ind} A$ the full subcategory of $\operatorname{mod} A$ formed by the indecomposable modules, and by D the standard duality $\operatorname{Hom}_K(-, J)$ on $\operatorname{mod} A$, where

E-mail address: skowyr@mat.uni.torun.pl.

http://dx.doi.org/10.1016/j.jalgebra.2014.06.006
0021-8693/© 2014 Elsevier Inc. All rights reserved.

J is a minimal injective cogenerator in $\operatorname{mod} K$. The Jacobson radical rad_A of $\operatorname{mod} A$ is the (two-sided) ideal generated by all noninvertible homomorphisms between modules in $\operatorname{ind} A$, and the infinite Jacobson radical $\operatorname{rad}_A^\infty$ of $\operatorname{mod} A$ is the intersection of all powers rad_A^i, $i \geqslant 1$, of rad_A. By a result due to M. Auslander [5] (see also [19] for an alternative proof), $\operatorname{rad}_A^\infty = 0$ if and only if A is of finite representation type, that is, $\operatorname{ind} A$ admits only a finite number of pairwise nonisomorphic modules. On the other hand, if A is of infinite representation type, then $(\operatorname{rad}_A^\infty)^2 \neq 0$, by a result proved in [14]. A module M in $\operatorname{mod} A$ is said to be sincere if every simple right A-module occurs as a composition factor of M, and faithful if its annihilator $\operatorname{ann}_A(M) = \{a \in A \mid Ma = 0\}$ vanishes. For a module X in $\operatorname{mod} A$ and its minimal projective presentation $P_1 \xrightarrow{f} P_0 \longrightarrow X \longrightarrow 0$ in $\operatorname{mod} A$, the transpose $\operatorname{Tr} X$ of X is the cokernel of the map $\operatorname{Hom}_A(f, A)$ in $\operatorname{mod} A^{\mathrm{op}}$, where A^{op} is the opposite algebra of A. The homological operator $\tau_A = D\operatorname{Tr}$ on modules in $\operatorname{mod} A$, called the Auslander–Reiten translation, is playing a fundamental role in the modern representation theory of algebras. Finally, following [10,16] a *tilted algebra* is an algebra of the form $\operatorname{End}_H(T)$, where H is a hereditary algebra and T is a (multiplicity-free) tilting module in $\operatorname{mod} H$, that is, $\operatorname{Ext}_H^1(T, T) = 0$ and the number of pairwise nonisomorphic indecomposable direct summands of T is equal to the rank of the Grothendieck group $K_0(H)$ of H.

The tilted algebras have for a long time played a central role in the representation theory of algebras and attracted much attention. In particular, the following handy criterion for an algebra to be a tilted algebra has been established independently by S. Liu [23] and A. Skowroński [37]: an algebra A is a tilted algebra if and only if the Auslander–Reiten quiver Γ_A of A admits a connected component with a faithful section Δ such that $\operatorname{Hom}_A(X, \tau_A Y) = 0$ for all modules X and Y in Δ. This was extended by I. Reiten and A. Skowroński to the double tilted algebras [30] and the generalized double tilted algebras [31], by defining double sections and multisections. The module category $\operatorname{mod} A$ of a generalized double tilted algebra A is determined, up to finitely many indecomposable modules, by the module categories of (left and right) tilted algebras naturally associated with A (see [31, Section 3] for details). We also mention that, for a generalized double tilted algebra A, all but finitely many modules X in $\operatorname{ind} A$ have projective dimension or injective dimension at most one, while A may be of an arbitrary (finite or infinite) global dimension. Note also that the class of generalized double tilted algebras was studied independently by I. Assem, F.U. Coelho, M. Lanzilotta and others under the name of strict laura algebras (see [1,11–13], for some results).

It has been proved by A. Jaworska, P. Malicki and A. Skowroński in [18] that an algebra A is a tilted algebra if and only if there exists a sincere module M in $\operatorname{mod} A$ such that, for any module X in $\operatorname{ind} A$, we have $\operatorname{Hom}_A(X, M) = 0$ or $\operatorname{Hom}_A(M, \tau_A X) = 0$. Recall that, by [6,32], a sequence $X \to M \to \tau_A X$ of nonzero homomorphisms in a module category $\operatorname{mod} A$ with X being indecomposable is called a short chain, and M the middle of this short chain. Therefore, an algebra A is a tilted algebra if and only if $\operatorname{mod} A$ admits a sincere module M which is not the middle of a short chain (affirmative

Download English Version:

https://daneshyari.com/en/article/4584676

Download Persian Version:

https://daneshyari.com/article/4584676

Daneshyari.com