Journal of Algebra 416 (2014) 287–313

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

S_n- and GL_n-module structures on free Novikov algebras

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A R T I C L E   I N F O

Article history:
Received 25 December 2013
Available online 15 July 2014
Communicated by Alberto Elduque

MSC:
17A50

Keywords:
Novikov algebra
Specht module
Weyl module

A B S T R A C T

An algebra with two identities $a(bc) = b(ac)$, $a(bc) - (ab)c = a(cb) - (ac)b$, is called Novikov. Module structures of a free Novikov algebra over permutation group and general linear group are studied.

1. Introduction

Construction of free algebras is one of the important problems of modern algebra. They appear in studying the varieties of algebras, polynomial identities and in operads theory. Multilinear parts of free algebras contain essential information about free algebras. Multilinear parts are studied by combinatorial methods and by methods of representation theory. In our paper we focus our attention on module structures of multilinear parts of free Novikov algebras. By module structures we mean modules over permutation group and over general linear group. Recall that any irreducible S_n-module

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http://dx.doi.org/10.1016/j.jalgebra.2014.06.013

can be characterized by a partition of n or by Young diagrams. The so-called Specht modules S^α, where $\alpha \vdash n$ is a partition of n, give us a complete list of irreducible modules of permutation group S_n. There exist deep connections between irreducible S_n-modules and irreducible $GL(V)$-modules. Any irreducible $GL(V)$-module is isomorphic to a so-called Weyl module. Any Weyl module as Specht module can be characterized by Young diagrams. These facts are known and one can find details, for example, in [7,8].

The list of algebraic varieties with studied free objects is given in [12]. Let us recall some of such results.

If F_n^{ass} is a multilinear part of a free associative algebra with n generators, then F_n^{ass}, as a module over permutations group S_n, is isomorphic to a regular module. Any irreducible S_n-module is associative admissible. This means that any irreducible S_n-module appears in decomposition of F_n^{ass} to a direct sum of irreducible components with non-zero multiplicity. Moreover, for any irreducible S_n-module its multiplicity in such decomposition is equal to dimension of this module.

If F_n^{lie} is a multilinear part of a free Lie algebra with $n \geq 3$ generators, then by [6] any irreducible S_n-module S^α, except when $\alpha = (1^n), (n), (2^2), (2^3)$, is a Lie admissible module. Multiplicity of such a module in decomposition of F_n^{lie} can be calculated in terms of major indices of Young diagrams. It was done in [9].

Similar questions for multilinear parts of free bicommutative algebras are studied in [5]. For degrees $n \leq 7$ module structures over S_n for multilinear parts of free anti-commutative algebras were found in [1].

The aim of our paper is to study multilinear parts of free Novikov algebras. We introduce a notion of Novikov weights and we describe irreducible components of multilinear parts in terms of weights. We find a criterion for Novikov admissible irreducible modules. We prove that multiplicities of irreducible components are ruled by Kostka numbers.

2. Statement of main result

Let *rsym* (right-symmetric polynomial) and *lcom* (left-commutative polynomial) be non-commutative non-associative polynomials defined by

$$rsym = t_1(t_2t_3) - t_1(t_3t_2) - (t_1t_2)t_3 + (t_1t_3)t_2,$$
$$lcom = t_1(t_2t_3) - t_2(t_1t_3).$$

An algebra with identities $rsym = 0$ and $lcom = 0$ is called *right-Novikov.* In our paper we consider only right-Novikov algebras therefore the word "right" will be omitted. So, if $A = (A, \circ)$ is a Novikov algebra with multiplication $a \circ b$, then

$$(a, b, c) = (a, c, b),$$
$$a \circ (b \circ c) = b \circ (a \circ c),$$

for any $a, b, c \in A$. Here