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Distributive lattices and the poset of pre-projective tilting modules

Ryoichi Kase

Department of Mathematics, Nara Women's University, Kitauoya-Nishimachi, Nara city, Nara, 630-8506, Japan

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ABSTRACT

D. Happel and L. Unger defined a partial order on the set of basic tilting modules. We study the poset of basic pre-projective tilting modules over path algebra of infinite type. We give an equivalent condition for that this poset is a distributive lattice. We also give an equivalent condition for that a distributive lattice is isomorphic to the poset of basic pre-projective tilting modules over path algebra of infinite type.

Introduction

Tilting theory first appeared in an article by Brenner and Butler [4]. In that article the notion of a tilting module for finite dimensional algebra was introduced. Tilting theory now appears in many areas of mathematics, for example algebraic geometry, theory of algebraic groups and algebraic topology. Let T be a tilting module for finite dimensional algebra A and let $B = \mathrm{End}_A(T)$. Happel showed that the two bounded derived categories $\mathcal{D}^{\mathrm{b}}(A)$ and $\mathcal{D}^{\mathrm{b}}(B)$ are equivalent as triangulated category. Therefore classifying tilting modules is an important problem.

E-mail address: r-kase@cc.nara-wu.ac.jp.

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Theory of tilting-mutation introduced by Riedtmann and Schofield is an approach to this problem [13]. They introduced the tilting quiver whose vertices are basic tilting modules and arrows correspond to mutation. Happel and Unger defined a partial order on the set of basic tilting modules and showed that the tilting quiver coincides with the Hasse quiver of this poset [7,8]. These combinatorial structures are now studied by many authors.

Notations. Let Q be a finite connected quiver without loops or oriented cycles. We denote by Q_0 (resp.Q_1) the set of vertices (resp. arrows) of Q. For any arrow $\alpha \in Q_1$ we denote by $s(\alpha)$ its starting point and denote by $t(\alpha)$ its target point (i.e. α is an arrow from $s(\alpha)$ to $t(\alpha)$). Let kQ be the path algebra of Q over an algebraically closed field k. Denote by mod-kQ the category of finite dimensional right kQ modules and by ind-kQ the category of indecomposable modules in mod-kQ. For any module $M \in$ mod-kQ, we denote by add M the category of all direct summands of finite direct sums of copies of M and denote by $|M|$ the number of pairwise non-isomorphic indecomposable direct summands of M. For any paths $w : a_0 \overset{\alpha_1}{\to} a_1 \overset{\alpha_2}{\to} \cdots \overset{\alpha_r}{\to} a_r$ and $w' : b_0 \overset{\beta_1}{\to} b_1 \overset{\beta_2}{\to} \cdots \overset{\beta_s}{\to} b_s$, the product is defined by

$$w \cdot w' := \begin{cases} a_0 \overset{\alpha_1}{\to} a_1 \overset{\alpha_2}{\to} \cdots \overset{\alpha_r}{\to} a_r = b_0 \overset{\beta_1}{\to} b_1 \overset{\beta_2}{\to} \cdots \overset{\beta_s}{\to} b_s & \text{if } a_r = b_0 \\ 0 & \text{if } a_r \neq b_0, \end{cases}$$

in kQ. Let $P(i)$ be an indecomposable projective module in mod-kQ associated with vertex $i \in Q_0$.

In this paper we will consider the set $\mathcal{T}_{\mathrm{p}}(Q)$ of basic pre-projective tilting modules and study its combinatorial structure.

The following result was shown in [9,11].

Theorem 0.1. *(See [9,11].) Let Q be a non-Dynkin quiver. Then the following are equivalent.*

(1) *All pre-projective tilting modules are slice modules.*
(2) *Q satisfies the following condition* (C)*:*

$$\text{(C)} \quad \delta(a) := \#\{\alpha \in Q_1 \mid s(\alpha) = a \text{ or } t(\alpha) = a\} \geq 2 \ \forall a \in Q_0,$$

where a module S is said to be a slice module if S is sincere and add S *satisfies the following conditions* (*see for example* [12, Section 4.2])*:*

(1) *If there is a path*

$$X = X_0 \to X_1 \to \cdots \to X_{r-1} \to X_r = Y$$