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The algebra of multitangent functions

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A B S T R A C T

Multizeta values are numbers appearing in many different contexts. Unfortunately, their arithmetics remains mostly out of reach.

In this article, we define a functional analogue of the algebra of multizeta values, namely the algebra of multitangent functions, which are 1-periodic functions defined by a process formally similar to multizeta values.

We introduce here the fundamental notions of reduction into monotangent functions, projection onto multitangent functions and that of trifactorization, giving a way of writing a multitangent function in terms of Hurwitz multizeta functions. This explains why the multitangent algebra is a functional analogue of the algebra of multizeta values. We then discuss the most important algebraic and analytic properties of these functions and their consequences on multizeta values, as well as their regularization in the divergent case.

Each property of multitangents has a pendant on the side of multizeta values. This allows us to propose new conjectures, which have been checked up to the weight 18.

Contents

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