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## The $\mathfrak{sl}_N$ -web algebras and dual canonical bases



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### ABSTRACT

In this paper, which is a follow-up to [38], I define and study  $\mathfrak{sl}_N$ -web algebras, for any  $N \geq 2$ . For  $N = 2$  these algebras are isomorphic to Khovanov’s [22] arc algebras and for  $N = 3$  they are Morita equivalent to the  $\mathfrak{sl}_3$ -web algebras which I defined and studied together with Pan and Tubbenhauer [34]. The main result of this paper is that the  $\mathfrak{sl}_N$ -web algebras are Morita equivalent to blocks of certain level- $N$  cyclotomic KLR algebras, for which I use the categorified quantum skew Howe duality defined in [38].

Using this Morita equivalence and Brundan and Kleshchev’s [4] work on cyclotomic KLR-algebras, I show that there exists an isomorphism between a certain space of  $\mathfrak{sl}_N$ -webs and the split Grothendieck group of the corresponding  $\mathfrak{sl}_N$ -web algebra, which maps the dual canonical basis elements to the Grothendieck classes of the indecomposable projective modules (with a certain normalization of their grading).

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## 1. Introduction

In [14] Cautis, Kamnitzer and Morrison defined  $\mathfrak{sl}_N$ -webs and the relations they satisfy, for arbitrary  $N \in \mathbb{N}_{\geq 2}$ . In [38] Yonezawa and I defined certain  $\mathfrak{sl}_N$ -web spaces  $W_\Lambda$  for arbitrary  $N \in \mathbb{N}_{\geq 2}$  and  $\Lambda := N\omega_\ell$ , where  $\omega_\ell$  is the  $\ell$ -th fundamental  $\mathfrak{sl}_m$ -weight with  $m = N\ell$  for arbitrary  $\ell \in \mathbb{N}$ . In the same paper, Cautis, Kamnitzer and Morrison also defined *quantum skew Howe duality*, which relates the representation theories of  $\dot{U}_q(\mathfrak{sl}_N)$  and  $\dot{U}_q(\mathfrak{sl}_m)$ . Using this duality, we obtained a  $\dot{U}_q(\mathfrak{sl}_m)$ -action on  $W_\Lambda$  and showed that there exists an isomorphism of  $\dot{U}_q(\mathfrak{sl}_m)$ -modules

$$V_\Lambda \cong W_\Lambda. \tag{1}$$

Here  $V_\Lambda$  is the irreducible  $\dot{U}_q(\mathfrak{sl}_m)$ -module of highest weight  $\Lambda$ , obtained as a quotient of the Verma module with the same highest weight.

In the same paper, we also defined  $\mathbb{C}$ -linear additive  $\mathfrak{sl}_N$ -web categories  $\mathcal{W}_\Lambda^\circ$ , using colored  $\mathfrak{sl}_N$ -matrix factorizations. We showed that  $\mathcal{W}_\Lambda^\circ$  is a strong  $\mathfrak{sl}_m$  2-representation and that there exists an equivalence of strong  $\mathfrak{sl}_m$  2-representations

$$\mathcal{V}_\Lambda^p \cong \dot{\mathcal{W}}_\Lambda^\circ. \tag{2}$$

Here  $\dot{\mathcal{W}}_\Lambda^\circ$  denotes the Karoubi envelope (i.e. idempotent completion) of  $\mathcal{W}_\Lambda^\circ$  and  $\mathcal{V}_\Lambda^p := R_\Lambda\text{-pmod}_{\text{gr}}$  is the category of finite-dimensional graded projective modules of the cyclotomic Khovanov–Lauda–Rouquier (KLR) algebra  $R_\Lambda$ . As we argued in [38], this result can be seen as a categorification of an instance of the quantum skew Howe duality defined in [14].

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