Journal of Algebra 409 (2014) 362–381

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

Equivariant quantization of Poisson homogeneous spaces and Kostant's problem

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ARTICLE INFO

Article history:
Received 2 February 2012
Available online 24 April 2014
Communicated by Louis Rowen

MSC:
17B37
17B10
53D17
53D55

Keywords:
Quantized universal enveloping algebra
Kostant's problem
Highest weight module
Equivariant quantization
Reduced fusion element

ABSTRACT

We find a partial solution to the longstanding problem of Kostant concerning description of the so-called locally finite endomorphisms of highest weight irreducible modules. The solution is obtained by means of its reduction to a far-reaching extension of the quantization problem. While the classical quantization problem consists in finding $\star$-product deformations of the commutative algebras of functions, we consider the case when the initial object is already a noncommutative algebra, the algebra of functions within q-calculus.

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[1] Supported in part by the Royal Swedish Academy of Sciences.
[2] Supported in part by NSF grant DMS-0901616.

http://dx.doi.org/10.1016/j.jalgebra.2014.03.033
0021-8693/

1. Introduction

The present paper completes a series of papers written by the authors on relations between Poisson homogeneous spaces and their quantization, solutions of the dynamical Yang–Baxter equation, and Kostant's problem [12–15].

Poisson homogeneous spaces were introduced by Drinfeld and their relations with the so-called classical doubles [1] were explained in [2]. Using that, the first author classified Poisson homogeneous spaces of quasi-triangular type in terms of Lagrangian subalgebras of $\mathfrak{g} \times \mathfrak{g}$, where $\mathfrak{g}$ is a semisimple complex finite-dimensional Lie algebra [11].

Later in [20], Lu discovered a strong similarity between classification of the Poisson homogeneous spaces of [11] and classification of trigonometric solutions of the classical dynamical Yang–Baxter equation (CDYBE) of [25]. This similarity was explained in [12]. Furthermore, the authors gave a classification of Poisson homogeneous spaces of triangular type, and, developing Lu's method, they proved that there is a natural one-to-one correspondence between Poisson homogeneous spaces of triangular type and rational solutions of the CDYBE.

The latter result opened a way for an explicit quantization of certain Poisson homogeneous spaces of triangular type using methods of [4] (one should notice that the existence of quantization of Poisson manifolds was proved by Kontsevich in [18]).

This idea was realized in [13–15] and first relations between quantization of Poisson homogeneous spaces of triangular type and the so-called Kostant's problem for the highest weight irreducible $U(\mathfrak{g})$-modules were considered in [15]. It is worth to mention that, in particular, this approach enabled the authors to obtain explicit formulas for quantization of the Kirillov–Kostant–Lie Poisson bracket on reductive co-adjoint orbits of $\mathfrak{g}$.

The main goal of the present paper is to develop methods for an explicit quantization of Poisson homogeneous spaces of quasi-triangular type and its relations with Kostant's problem.

It turns out that in order to get explicit formulas in the quasi-triangular case one has to work instead of the standard universal enveloping algebra of $\mathfrak{g}$ and the algebra of regular functions on the corresponding Lie group G with their quantized versions. As a consequence, it turns out that the latter quantization problems are related to Kostant's problem for the quantum universal enveloping algebra.

To be more precise, let $\check{U}_q\mathfrak{g}$ be the quantized universal enveloping algebra "of simply connected type" [9] that corresponds to a finite-dimensional semisimple Lie algebra $\mathfrak{g}$. Let $L(\lambda)$ be the irreducible highest weight $\check{U}_q\mathfrak{g}$-module of highest weight λ. In this paper we show that for certain values of λ, the action map $\check{U}_q\mathfrak{g} \to (\operatorname{End} L(\lambda))_{\text{fin}}$ is surjective. Here $(\operatorname{End} L(\lambda))_{\text{fin}}$ stands for the locally finite part of $\operatorname{End} L(\lambda)$ with respect to the adjoint action of $\check{U}_q\mathfrak{g}$. For the Lie-algebraic case $q = 1$, this problem is known as the classical Kostant's problem, see, e.g., [7,8,21,22]. The complete answer to it is still unknown even in the $q = 1$ case. However, there are examples of λ for which the action