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Quiver Schur algebras and Koszul duality



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ABSTRACT

Stroppel and Webster have introduced a grading on the cyclotomic q -Schur algebra S_d^s . We prove that the obtained graded algebra is graded Morita equivalent to a Koszul algebra. The proof is based on a result of Rouquier, Shan, Varagnolo and Vasserot that identifies the category $\text{mod}(S_d^s)$ with a subcategory of an affine parabolic category \mathcal{O} of type A. This subcategory admits a Koszul grading constructed by Shan, Varagnolo and Vasserot. We identify this Koszul grading with the grading on $\text{mod}(S_d^s)$.

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1. Introduction

Let e, l be integers, $e > 1, l > 0$. Let Γ be the cyclic quiver of type \widehat{A}_{e-1} and let I be the set of its vertices. Let $q = \zeta \in \mathbb{C}$ be a primitive e th root of unity. With each $d \in \mathbb{N}$ and each tuple $\mathbf{s} \in (\mathbb{Z}/e\mathbb{Z})^l$, Dipper, James and Mathas associate a cyclotomic q -Schur \mathbb{C} -algebra $S_{\mathbf{d}}^{\mathbf{s}}$ in [8]. This algebra admits a block decomposition $S_{\mathbf{d}}^{\mathbf{s}} = \bigoplus_{\mathbf{d}} S_{\mathbf{d}}^{\mathbf{s}}$, where \mathbf{d} runs over the set of elements $\mathbf{d} = \sum_{i \in I} d_i \cdot i$ in $\mathbb{N}I$ such that $\sum_{i \in I} d_i = d$. For each \mathbf{d} , the block $S_{\mathbf{d}}^{\mathbf{s}}$ is quasi-hereditary and the standard modules are the so-called *Weyl modules*. They are labeled by a subset $\mathcal{P}_{\mathbf{d}}^{\mathbf{s}}$ of the set \mathcal{P}_d^l of l -partitions of d .

In Section 3.1 we define a class of gradings on $S_{\mathbf{d}}^{\mathbf{s}}$ called *admissible gradings*. An example of such grading is given in [30]. Fixing an admissible grading of $S_{\mathbf{d}}^{\mathbf{s}}$ yields a grading of the category $\text{mod}(S_{\mathbf{d}}^{\mathbf{s}})$.

By [26], we can identify the category $\text{mod}(S_{\mathbf{d}}^{\mathbf{s}})$ of finite dimensional $S_{\mathbf{d}}^{\mathbf{s}}$ -modules with a highest weight subcategory \mathbf{A} of a parabolic category \mathcal{O} of $\widehat{\mathfrak{gl}}_N$ at negative level (for some positive integer N), see Theorem 3.13. The category \mathbf{A} has a Koszul grading by [28, Thm. 6.4]. It is natural to ask whether the grading on $\text{mod}(S_{\mathbf{d}}^{\mathbf{s}})$ above coincides with the Koszul grading of \mathbf{A} .

In Section 3.9 we define a basic graded algebra ${}^b S_{\mathbf{d}}^{\mathbf{s}}$ which is graded Morita equivalent to $S_{\mathbf{d}}^{\mathbf{s}}$. Let $S^{\mathcal{O}}$ be the endomorphism algebra of the minimal projective generator of \mathbf{A} (equipped with the Koszul grading). Our main result is the following theorem, see Theorem 3.40 below.

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