Journal of Algebra 406 (2014) 91–133

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

Quiver Schur algebras and Koszul duality

Ruslan Maksimau

Université Paris Diderot, Paris 7, Institut de Mathématiques de Jussieu, Paris Rive Gauche, UMR7586, Bâtiment Sophie Germain, Case 7012, 75205 Paris Cedex 13, France

A R T I C L E   I N F O

Article history:
Received 11 August 2013
Available online 18 March 2014
Communicated by Volodymyr Mazorchuk

Keywords:
Highest weight categories
Hecke algebras
Koszul duality
Parabolic category $\mathcal{O}$
q-Schur algebras

A B S T R A C T

Stroppel and Webster have introduced a grading on the cyclotomic q-Schur algebra $S_d^{\mathbf{s}}$. We prove that the obtained graded algebra is graded Morita equivalent to a Koszul algebra. The proof is based on a result of Rouquier, Shan, Varagnolo and Vasserot that identifies the category $\mathrm{mod}(S_d^{\mathbf{s}})$ with a subcategory of an affine parabolic category $\mathcal{O}$ of type A. This subcategory admits a Koszul grading constructed by Shan, Varagnolo and Vasserot. We identify this Koszul grading with the grading on $\mathrm{mod}(S_d^{\mathbf{s}})$.

Contents

E-mail addresses: ruslmax@gmail.com, maksimau@math.jussieu.fr.

http://dx.doi.org/10.1016/j.jalgebra.2014.02.029
0021-8693/

1. Introduction

Let e, l be integers, $e > 1$, $l > 0$. Let Γ be the cyclic quiver of type $\widehat{A}_{e-1}$ and let I be the set of its vertices. Let $q = \zeta \in \mathbb{C}$ be a primitive eth root of unity. With each $d \in \mathbb{N}$ and each tuple $\mathbf{s} \in (\mathbb{Z}/e\mathbb{Z})^l$, Dipper, James and Mathas associate a cyclotomic q-Schur $\mathbb{C}$-algebra $S_d^{\mathbf{s}}$ in [8]. This algebra admits a block decomposition $S_d^{\mathbf{s}} = \bigoplus_{\mathbf{d}} S_{\mathbf{d}}^{\mathbf{s}}$, where $\mathbf{d}$ runs over the set of elements $\mathbf{d} = \sum_{i\in I} d_i \cdot i$ in $\mathbb{N}I$ such that $\sum_{i\in I} d_i = d$. For each $\mathbf{d}$, the block $S_{\mathbf{d}}^{\mathbf{s}}$ is quasi-hereditary and the standard modules are the so-called *Weyl modules*. They are labeled by a subset $\mathcal{P}_{\mathbf{d}}^l$ of the set $\mathcal{P}_d^l$ of l-partitions of d.

In Section 3.1 we define a class of gradings on $S_{\mathbf{d}}^{\mathbf{s}}$ called *admissible gradings*. An example of such grading is given in [30]. Fixing an admissible grading of $S_{\mathbf{d}}^{\mathbf{s}}$ yields a grading of the category $\mathrm{mod}(S_{\mathbf{d}}^{\mathbf{s}})$.

By [26], we can identify the category $\mathrm{mod}(S_{\mathbf{d}}^{\mathbf{s}})$ of finite dimensional $S_{\mathbf{d}}^{\mathbf{s}}$-modules with a highest weight subcategory $\mathbf{A}$ of a parabolic category $\mathcal{O}$ of $\widehat{\mathfrak{gl}_N}$ at negative level (for some positive integer N), see Theorem 3.13. The category $\mathbf{A}$ has a Koszul grading by [28, Thm. 6.4]. It is natural to ask whether the grading on $\mathrm{mod}(S_{\mathbf{d}}^{\mathbf{s}})$ above coincides with the Koszul grading of $\mathbf{A}$.

In Section 3.9 we define a basic graded algebra ${}^bS_{\mathbf{d}}^{\mathbf{s}}$ which is graded Morita equivalent to $S_{\mathbf{d}}^{\mathbf{s}}$. Let $S^{\mathcal{O}}$ be the endomorphism algebra of the minimal projective generator of $\mathbf{A}$ (equipped with the Koszul grading). Our main result is the following theorem, see Theorem 3.40 below.