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Covering theory for linear categories with application to derived categories ☆

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ABSTRACT

We extend the Galois covering theory introduced by Bongartz–Gabriel for skeletal linear categories to general linear categories. We show that a Galois covering between Krull–Schmidt categories preserves irreducible morphisms and almost splits sequences. Specializing to derived categories, we study when a Galois covering between locally bounded linear categories induces a Galois covering between the bounded derived categories of finite dimensional modules. As an application, we show that each locally bounded linear category with radical squared zero admits a gradable Galois covering, which induces a Galois covering between the bounded derived categories of finite dimensional modules, and a Galois covering between the Auslander–Reiten quivers of these bounded derived categories. In a future paper, this will enable us to obtain a complete description of the bounded derived category of finite dimensional modules over a finite dimensional algebra with radical squared zero.

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Introduction

The covering technique has been playing an important role in the representation theory of finite dimensional algebras; see, for example, [6,8,9,16]. In this connection, algebras are regarded as locally bounded linear categories; see [6]. To each Galois covering between such categories, Bongartz–Gabriel associated a push-down functor between their module categories, which induces a Galois covering between the Auslander–Reiten quivers in the locally representation-finite case; see [6,8]. This technique was extended later by Asashiba by studying the induced push-down functor between the bounded homotopy categories of finitely generated projective modules; see [1]. Now, the push-down functor also induces an exact functor between the bounded derived categories of finite dimensional modules. It is then natural to ask when this derived push-down functor is a Galois covering. Unfortunately, this question is somehow problematic, since Gabriel's notion of a Galois covering is only for skeletal linear categories. To overcome this difficulty, Asashiba introduced the notion of a precovering and called a dense precovering a *covering*; see [2]. Strengthening this notion of a covering, we obtain the notion of a Galois covering for general linear categories. As an interesting example, the bounded derived category of finite dimensional representations of a finite acyclic quiver is a Galois covering of the corresponding cluster category introduced in [7]. One of the nice properties of such a Galois covering is that it preserves the Auslander–Reiten theory in case the categories are Krull–Schmidt. Most importantly, this provides a useful tool for studying the bounded derived category of finite dimensional modules over a locally bounded linear category. To give more details, we outline the content section by section.

In Section 1, we shall deal with the problem as to when the derived category of an abelian category has arbitrary direct sums. For this purpose, we introduce the notion of essential direct sums and show that if an abelian category has essential direct sums, then its derived category has direct sums. In particular, the derived category of all modules over a locally bounded linear category has direct sums.

In Section 2, based on Asashiba's notion of a precovering, we first define the notion of a Galois covering for general linear categories, and then search for conditions for a precovering to be a Galois covering. Moreover, we introduce the notion of a graded adjoint pair between linear categories, and show that restricting such an adjoint pair to appropriate subcategories yields precoverings.

In Section 3, we show that a Galois covering between two Krull–Schmidt categories preserves irreducible morphisms and almost split sequences. In particular, one of these categories has almost split sequences if and only the other one does.

In Section 4, we introduce the notion of a Galois covering for valued translation quivers, and show that a Galois covering between Hom-finite Krull–Schmidt categories induces a Galois covering of their Auslander–Reiten quivers.

In Sections 5, we shall strengthen Milicic's result that an adjoint pair of exact functors between abelian categories induces an adjoint pair between their derived categories;