Journal of Algebra 400 (2014) 43–55

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

The class of $(1,3)$-groups with homocyclic regulator quotient of exponent p^4 has bounded representation type

David M. Arnold [a,1], Adolf Mader [b], Otto Mutzbauer [c,*,2], Ebru Solak [d]

[a] Dept. of Math., Baylor University, Waco, TX 76798-7328, United States
[b] Dept. of Math., University of Hawaii, 2565 McCarthy Mall, Honolulu, HI 96822, United States
[c] Universität Würzburg, Math. Inst., Emil-Fischer-Str. 30, 97074 Würzburg, Germany
[d] Dept. of Math., Middle East Technical University, Inönü Bulvarı, 06531 Ankara, Turkey

ARTICLE INFO

Article history:
Received 16 December 2012
Available online 20 December 2013
Communicated by Gernot Stroth

MSC:
20K15
20K25
20K35
15A21
16G60

Keywords:
Almost completely decomposable group
Indecomposable
Bounded representation type

ABSTRACT

The class of almost completely decomposable groups with a critical typeset of type $(1,3)$ and a homocyclic regulator quotient of exponent p^4 is shown to be of bounded representation type. There are only nine near-isomorphism types of indecomposables, all of rank $\leqslant 6$.

1. Introduction

Representations of a finite partially ordered set (poset) S over a field F, denoted by $\mathrm{rep}(S,F)$, have been extensively investigated, [10]. The representation type of $\mathrm{rep}(S,F)$ has been determined

* Corresponding author.
E-mail addresses: David_Arnold@baylor.edu (D.M. Arnold), adolf@math.hawaii.edu (A. Mader), mutzbauer@mathematik.uni-wuerzburg.de (O. Mutzbauer), esolak@metu.edu.tr (E. Solak).

1 Research of the first author supported, in part, by funds from the University Research Committee and the Vice Provost for Research at Baylor University.
2 The third author would like to thank the Department of Mathematics at the University of Hawaii for the kind hospitality afforded to him.

0021-8693/$ – see front matter
http://dx.doi.org/10.1016/j.jalgebra.2013.10.029

in terms of S, independent of the field F, and indecomposables have been classified in the case of bounded representation type.

Much less is known about representations of posets S over $\mathbb{Z}_{p^k} = \mathbb{Z}/p^k\mathbb{Z}$. Just as for fields, these representations correspond to modules over the $\mathbb{Z}_{p^k}$-incidence algebra of S. The representation type of $\mathrm{rep}_{\mathrm{fg}}(S, \mathbb{Z}_{p^k})$ has been completely characterized in terms of S and m, [2], where objects of $\mathrm{rep}_{\mathrm{fg}}(S, \mathbb{Z}_{p^k})$ are representations $(U_0, U_s\colon\ s \in S)$ such that U_0 is a finitely generated $\mathbb{Z}_{p^k}$-module, each U_s is a submodule of U_0, and $U_s \subseteq U_t$ if $s \leqslant t$ in S. For most cases of S and m the representation type is unbounded so that a classification of indecomposables is not feasible.

The subclass $\mathrm{hcdrep}(S^*, \mathbb{Z}_{p^k})$ of $\mathrm{rep}_{\mathrm{fg}}(S^*, \mathbb{Z}_{p^k})$, where S^* is the disjoint union of S and $\{*\}$, consists of representations $(U_0, U_s\colon\ s \in S^*)$ such that U_0 is a finitely generated free $\mathbb{Z}_{p^k}$-module, $U_0 = \sum_{i\in S} U_i$, each U_i is a summand of U_0, U_s is a summand of U_t if $s \leqslant t$ in S, U_* is a summand of U_0, and $U_s \cap U_* = 0$ for each $s \in S$. Interest in $\mathrm{hcdrep}(S^*, \mathbb{Z}_{p^k})$ for abelian group theorists is motivated by the existence of a bijection from isomorphism classes of indecomposable representations in $\mathrm{hcdrep}(S^*, \mathbb{Z}_{p^k})$ to near-isomorphism classes of indecomposable homocyclic (S^{op}, p^k)-groups as defined in Section 3, S^{op} the opposite of a poset S of types, [4].

Some classes are shown to be of bounded representation type for homocyclic (S, p^k)-groups, hence for $\mathrm{hcdrep}(S^*, \mathbb{Z}_{p^k})$, provided by classifications of indecomposable (S, p^k)-groups given in [3,8,9], and [11] for $S = (1, 2)$ or $(1, 3)$ and various values of m where $(1, n)$ is the poset consisting of a disjoint union of a type and a chain of types of length n. This paper is devoted to a classification of indecomposable homocyclic $((1, 3), p^4)$-groups, thereby confirming bounded representation type in this case. More precisely, we present a complete collection of near-isomorphism types of indecomposables. They are of rank 4, 5, 6. The proof includes finding a normal form for an integer matrix, called a coordinate matrix, for a group, see Section 3.

Still unresolved are the cases $S = (1, n)$, $n \geqslant 4$, $m = 3$ and $S = (1, 3)$, $m = 5$.

2. Matrices

A *line* of a matrix is a row or a column. *Transformations* of matrices are understood to be successive applications of elementary transformations. While annihilating an entry, other entries that were originally zero may become non-zero; such entries are called *fill-ins*. A *unit* in our context is an integer that is relatively prime to p, and entries that are 0 modulo p^k are briefly said to be 0. An integer matrix $A = [a_{i,j}]$ is called *p-reduced* (*modulo p^k*) if

(1) there is at most one 1 in a line and all other entries are in $p\mathbb{Z}$,
(2) if an entry 1 of A is at the position (i_s, j_s), then $a_{i_s,j} = 0$ for all $j > j_s$ and $a_{i,j_s} = 0$ for all $i < i_s$, and $a_{i_s,j}, a_{i,j_s} \in p\mathbb{Z}$ for all $j < j_s$ and all $i > i_s$.

Thus in a p-reduced matrix, the entries left of and below an entry 1 are in $p\mathbb{Z}$.

Lemma 1. *(Cf. [3, Lemma 14].) Let A be an integer matrix.*

(1) *A can be transformed into a p-reduced matrix by elementary row transformations upward and elementary column transformations to the right.*
(2) *A can be transformed into a p-reduced matrix where all entries are 0 below a 1 by arbitrary elementary row transformations and elementary column transformations to the right.*

0-*lines may be moved by row and column interchanges, but they remain* 0.

Proof. (1). Let A be an integer matrix. We choose the left most column containing a unit u. If there are several units, choose the lowest one. Multiplying the column by u^{-1} (modulo p^k) we get the "pivot" entry 1 (a non-zero entry that will be used to annihilate) below which there are only entries in $p\mathbb{Z}$. With the pivot 1 we can achieve entries equal to 0 above and to the right of 1. We consider the next non-zero column that contains a unit and choose the unit u with the largest row index. Again we change this unit to 1, the new pivot. This pivot 1 is on a different row than the first pivot