Journal of Algebra 393 (2013) 92–101

Contents lists available at SciVerse ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

Asymptotics of H-identities for associative algebras with an H-invariant radical ☆

A.S. Gordienko

Vrije Universiteit Brussel, Belgium

ARTICLE INFO

Article history:
Received 6 December 2012
Available online 31 July 2013
Communicated by Louis Rowen

MSC:
primary 16R10
secondary 16R50, 16W20, 16W22, 16W25, 16T05, 20C30

Keywords:
Associative algebra
Polynomial identity
Derivation
Group action
Hopf algebra
H-module algebra
Codimension
Cocharacter
Young diagram

ABSTRACT

We prove the existence of the Hopf PI-exponent for finite dimensional associative algebras A with a generalized Hopf action of an associative algebra H with 1 over an algebraically closed field of characteristic 0 assuming only the invariance of the Jacobson radical $J(A)$ under the H-action and the existence of the decomposition of $A/J(A)$ into the sum of H-simple algebras. As a consequence, we show that the analog of Amitsur's conjecture holds for G-codimensions of finite dimensional associative algebras over a field of characteristic 0 with an action of an arbitrary group G by automorphisms and anti-automorphisms and for differential codimensions of finite dimensional associative algebras with an action of an arbitrary Lie algebra by derivations.

© 2013 Elsevier Inc. All rights reserved.

1. Introduction

Amitsur's conjecture on asymptotic behaviour of codimensions of ordinary polynomial identities was proved by A. Giambruno and M.V. Zaicev [10, Theorem 6.5.2] in 1999.

When an algebra is endowed with a grading, an action of a group G by automorphisms and anti-automorphisms, an action of a Lie algebra by derivations or a structure of an H-module algebra for some Hopf algebra H, it is natural to consider, respectively, graded, G-, differential or

☆ Supported by Fonds Wetenschappelijk Onderzoek—Vlaanderen Pegasus Marie Curie post doctoral fellowship (Belgium) and RFBR grant 13-01-00234a (Russia).

E-mail address: alexey.gordienko@vub.ac.be.

0021-8693/$ – see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.jalgebra.2013.05.032

H-identities [4–6,14]. The analog of Amitsur's conjecture for finite dimensional associative algebras with a $\mathbb{Z}_2$-action was proved by A. Giambruno and M.V. Zaicev [10, Theorem 10.8.4] in 1999. In 2010–2011, E. Aljadeff, A. Giambruno, and D. La Mattina [1,2,9] obtained the validity of the analog of Amitsur's conjecture for associative PI-algebras with an action of a finite Abelian group by automorphisms as a particular case of their result for graded algebras.

In 2012, the analog of the conjecture was proved [12,13] for finite dimensional associative algebras with a rational action of a reductive affine algebraic group by automorphisms and anti-automorphisms, with an action of a finite dimensional semisimple Lie algebra by derivations or an action of a semisimple Hopf algebra. These results were obtained as a consequence of [12, Theorem 5] and [13, Theorem 6], where the authors considered finite dimensional associative algebras with a generalized Hopf action of an associative algebra H with 1. In the proof, they required the existence of an H-invariant Wedderburn–Mal'cev and Wedderburn–Artin decompositions. Here we remove the first restriction. This enables us to prove the analog of Amitsur's conjecture for G-codimensions of finite dimensional associative algebras with an action of an arbitrary group G by automorphisms and anti-automorphisms and for differential codimensions of finite dimensional associative algebras with an action of an arbitrary Lie algebra by derivations.

2. Polynomial *H*-identities and their codimensions

Let H be a Hopf algebra over a field F. An algebra A over F is an *H-module algebra* or an *algebra with an H-action*, if A is endowed with a homomorphism $H \to \mathrm{End}_F(A)$ such that $h(ab) = (h_{(1)}a)(h_{(2)}b)$ for all $h \in H$, $a, b \in A$. Here we use Sweedler's notation $\Delta h = h_{(1)} \otimes h_{(2)}$ where Δ is the comultiplication in H.

In order to embrace an action of a group by anti-automorphisms, we consider a generalized Hopf action [6, Section 3].

Let H be an associative algebra with 1 over F. We say that an associative algebra A is an algebra with a *generalized H-action* if A is endowed with a homomorphism $H \to \mathrm{End}_F(A)$ and for every $h \in H$ there exist $h'_i, h''_i, h'''_i, h''''_i \in H$ such that

$$h(ab) = \sum_i \big(\big(h'_i a\big)\big(h''_i b\big) + \big(h'''_i b\big)\big(h''''_i a\big)\big) \quad \text{for all } a, b \in A. \tag{1}$$

Choose a basis $(\gamma_\beta)_{\beta \in \Lambda}$ in H and denote by $F\langle X|H\rangle$ the free associative algebra over F with free formal generators $x_i^{\gamma_\beta}$, $\beta \in \Lambda$, $i \in \mathbb{N}$. Let $x_i^h := \sum_{\beta \in \Lambda} \alpha_\beta x_i^{\gamma_\beta}$ for $h = \sum_{\beta \in \Lambda} \alpha_\beta \gamma_\beta$, $\alpha_\beta \in F$, where only finite number of α_β are nonzero. Here $X := \{x_1, x_2, x_3, \ldots\}$, $x_j := x_j^1$, $1 \in H$. We refer to the elements of $F\langle X|H\rangle$ as H-polynomials. Note that here we do not consider any H-action on $F\langle X|H\rangle$.

Let A be an associative algebra with a generalized H-action. Any map $\psi : X \to A$ has a unique homomorphic extension $\bar\psi : F\langle X|H\rangle \to A$ such that $\bar\psi(x_i^h) = h\psi(x_i)$ for all $i \in \mathbb{N}$ and $h \in H$. An H-polynomial $f \in F\langle X|H\rangle$ is an *H-identity* of A if $\bar\psi(f) = 0$ for all maps $\psi : X \to A$. In other words, $f(x_1, x_2, \ldots, x_n)$ is an H-identity of A if and only if $f(a_1, a_2, \ldots, a_n) = 0$ for any $a_i \in A$. In this case we write $f \equiv 0$. The set $\mathrm{Id}^H(A)$ of all H-identities of A is an ideal of $F\langle X|H\rangle$. Note that our definition of $F\langle X|H\rangle$ depends on the choice of the basis $(\gamma_\beta)_{\beta \in \Lambda}$ in H. However such algebras can be identified in the natural way, and $\mathrm{Id}^H(A)$ is the same.

Denote by P_n^H the space of all multilinear H-polynomials in $x_1, \ldots, x_n$, $n \in \mathbb{N}$, i.e.

$$P_n^H = \big\langle x^{h_1}_{\sigma(1)} x^{h_2}_{\sigma(2)} \ldots x^{h_n}_{\sigma(n)} \bigm| h_i \in H,\ \sigma \in S_n \big\rangle_F \subset F\langle X|H\rangle.$$

Then the number $c_n^H(A) := \dim(\frac{P_n^H}{P_n^H \cap \mathrm{Id}^H(A)})$ is called the nth *codimension of polynomial H-identities* or the nth *H-codimension* of A.

The analog of Amitsur's conjecture for H-codimensions of A can be formulated as follows.

Download English Version:

https://daneshyari.com/en/article/4585011

Download Persian Version:

https://daneshyari.com/article/4585011

Daneshyari.com