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Journal of Algebra 299 (2006) 503–534

JOURNAL OF
Algebra

www.elsevier.com/locate/jalgebra

Toric singularities revisited

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Received 19 March 2005

Available online 12 July 2005

Communicated by Steven Dale Cutkosky

Abstract

In [K. Kato, Toric singularities, Amer. J. Math. 116 (5) (1994) 1073–1099], Kato defined his notion of a log regular scheme and studied the local behavior of such schemes. A toric variety equipped with its canonical logarithmic structure is log regular. And, these schemes allow one to generalize toric geometry to a theory that does not require a base field. This paper will extend this theory by removing normality requirements.
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Keywords: Toric varieties; Semigroup rings

Conventions and notation

All monoids considered in this paper are commutative and cancellative. All rings considered in this paper are commutative and unital. See Kato [2] for an introduction to log schemes. There Kato defines pre-log structures and log structures on the étale site of X. However, we will use the Zariski topology throughout this paper.

P^*	the unit group of the monoid P.
$\overline{P}$	the sharp image of the monoid P, $\overline{P} = P/P^*$ is the orbit space under the natural action of P^* on P.
P^+	$P^+ = P \setminus P^*$ is the maximal ideal of the monoid P.

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doi:10.1016/j.jalgebra.2005.05.029

P^{gp} the group generated by P, that is, image of P under the left adjoint of the inclusion functor from Abelian groups to monoids.
P^{sat} the saturation of P, that is, $\{p \in P^{\mathrm{gp}} \mid np \in P$ for some $n \in \mathbb{N}^+\}$.
$R[P]$ the monoid algebra of P over a ring R. The elements of $R[P]$ are written as "polynomials". That is, they are finite sums $\sum r_p t^p$ with coefficients in R and exponents in P.
(K) the ideal $\beta(K)A$, where $\beta: P \to A$ is a monoid homomorphism with respect to multiplication on A and K is an ideal of P. We say such an ideal is a *log ideal* of A.
$R[\![P]\!]$ the (P^+)-adic completion of $R[P]$.
$P_{\mathfrak{p}}$ the localization of the monoid P at the prime ideal $\mathfrak{p} \subseteq P$.
$\dim P$ the (Krull) dimension of the monoid P.

Introduction

A toric variety is a normal irreducible separated scheme X, locally of finite type over a field k, which contains an algebraic torus $T \cong (k^*)^d$ as an open set and is endowed with an algebraic action $T \times X \to X$ extending the group multiplication $T \times T \to T$. According to Oda [3]:

> The theory was started at the beginning of 1970s by Demazure [4] in connection with algebraic subgroups of the Cremona groups, by Mumford et al. [5] and Satake [6] in connection with compactifications of locally symmetric varieties, and by Miyake and Oda [7]. We were inspired by Hochster [8] as well as Sumihiro [9,10].
>
> Comprehensive surveys from various different perspectives can be found in Danilov [11], Mumford et al. [5,12] as well as [13,14].

In [1], Kato extended the theory of toric geometry over a field to an absolute theory, without base. This is achieved by replacing the notion of a toroidal embedding introduced in [5] with the notion of a log structure. A toroidal embedding is a pair (X, U) consisting of a scheme X locally of finite type and an open subscheme $U \subset X$ such that (X, U) is isomorphic, locally in the étale topology, to a pair consisting of a toric variety and its algebraic torus. Toroidal embeddings are particularly nice locally Noetherian schemes with distinguished log structures.

A log structure on a scheme X, in the sense of Fontaine and Illusie, is a morphism of sheaves of monoids $\alpha: \mathcal{M}_X \to \mathcal{O}_X$ restricting to an isomorphism $\alpha^{-1}(\mathcal{O}_X^*) \cong \mathcal{O}_X^*$. The theory of log structures on schemes is developed by Kato in [2]. Log structures were developed to give a unified treatment of the various constructions of de Rham complexes with logarithmic poles. In [15] Illusie recalls the question that motivated their definition:

> Let me briefly recall what the main motivating question was. Suppose S is the spectrum of a complete discrete valuation ring A, with closed (respectively generic) point s (respectively η), and X/S is a scheme with semi-stable reduction, which means that, locally for the étale topology, X is isomorphic to the closed subscheme of $\mathbb{A}^n_S$ defined by

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