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A conjecture for q-decomposition matrices of cyclotomic v-Schur algebras

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Abstract

The Jantzen sum formula for cyclotomic v-Schur algebras yields an identity for some q-analogues of the decomposition matrices of these algebras. We prove a similar identity for matrices of canonical bases of higher-level Fock spaces. We conjecture then that those matrices are actually identical for a suitable choice of parameters. In particular, we conjecture that decomposition matrices of cyclotomic v-Schur algebras are obtained by specializing at $q = 1$ some transition matrices between the standard basis and the canonical basis of a Fock space.

Keywords: Decomposition matrices; Cyclotomic Schur algebras; Canonical bases; Fock spaces

1. Introduction

In order to study representations of the Ariki–Koike algebra associated to the complex reflection group $G(l, 1, m)$, Dipper, James and Mathas introduced in 1998 the cyclotomic v-Schur algebra [DJM]. This algebra depends on the two integers l and m and on some deformation parameters $v, u_1, \ldots, u_l$. When $l = 1$, the cyclotomic v-Schur algebra coincides with the v-Schur algebra of [DJ]. It is an open problem to calculate the decomposition matrix of a cyclotomic v-Schur algebra whose parameters are powers of a given nth root of unity. To this aim, James and Mathas proved, for cyclotomic v-Schur algebras, an important formula: the Jantzen sum formula [JM]. Given a Jantzen filtration for Weyl modules, one can define a q-analogue $D(q)$ of the decomposition matrix; the coefficients of $D(q)$ are graded decomposition numbers of the

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composition factors of Weyl modules (see Definition 2.5). The Jantzen sum formula is equivalent to the identity $D'(1) = J^{\lhd} D(1)$, where $J^{\lhd}$ is a matrix of $\wp$-adic valuations of factors of some Gram determinants (see Theorem 2.3 and Corollary 2.7).

Let $\Delta(q)$ be the matrix of the canonical basis of the degree m homogeneous component of a Fock representation of level l of $U_q(\widehat{\mathfrak{sl}}_n)$ [U2]. Uglov provided in [U2] an algorithm for computing $\Delta(q)$.

In view of Ariki's theorem for Ariki–Koike algebras [A2], it seems natural to conjecture that for a suitable choice of parameters, one has $D(q) = \Delta(q)$. This would provide an algorithm for computing decomposition matrices of cyclotomic v-Schur algebras. Varagnolo and Vasserot [VV] proved for $l = 1$ that $D(1) = \Delta(1)$. Moreover, Ryom-Hansen showed that this conjecture (still for $l = 1$) is compatible with the Jantzen–Schaper formula [Ry]. Passing to higher level $l \geqslant 1$ requires the introduction of an extra parameter $\mathbf{s}_l = (s_1, \dots, s_l) \in \mathbb{Z}^l$, called *multi-charge*; this l-tuple parametrizes the Fock space of level l introduced by Uglov. We say that $\mathbf{s}_l$ is *m-dominant* if for all $1 \leqslant d \leqslant l-1$, we have $s_{d+1} - s_d \geqslant m$. In this case, we conjecture that $D(q) = \Delta(q)$. Here, $D(q)$ comes from a Jantzen filtration of the Weyl modules of the cyclotomic v-Schur algebra $\mathcal{S}_{\mathbb{C}} = \mathcal{S}_{\mathbb{C},m}(\zeta; \zeta^{s_1}, \dots, \zeta^{s_l})$ with $\zeta := \exp(\frac{2i\pi}{n})$. Note that for any choice of roots of unity $\zeta^{\mathtt{r}_1}, \dots, \zeta^{\mathtt{r}_l}$ (that is, for any $\mathtt{r}_1, \dots, \mathtt{r}_l \in \mathbb{Z}/n\mathbb{Z}$) and any m we can find an m-dominant multi-charge $\mathbf{s}_l = (s_1, \dots, s_l)$ such that $\zeta^{s_d} = \zeta^{\mathtt{r}_d}$ $(1 \leqslant d \leqslant l)$. Therefore, putting $q = 1$, our conjecture gives an algorithm for calculating the decomposition matrix of an arbitrary cyclotomic v-Schur algebra $\mathcal{S}_{\mathbb{C}} = \mathcal{S}_{\mathbb{C},m}(\zeta; \zeta^{s_1}, \dots, \zeta^{s_l})$. Such a conjecture is new even for type B_m (case $l = 2$).

Our conjecture is supported by the following theorem. We define in a combinatorial way a matrix $J^{\prec}$ for any multi-charge $\mathbf{s}_l$; if $\mathbf{s}_l$ is m-dominant, then our matrix $J^{\prec}$ coincides with the matrix $J^{\lhd}$ of the Jantzen sum formula. We show then that for any multi-charge $\mathbf{s}_l$, we have $\Delta'(1) = J^{\prec} \Delta(1)$ (Theorem 2.8).

The proof of our theorem relies on a combinatorial expression for the derivative at $q = 1$ of the matrix $A(q)$, where $A(q)$ is the matrix of the Fock space involution used for defining $\Delta(q)$. Namely, we show that $A'(1) = 2J^{\prec}$ (Theorem 2.11). The coefficients of $A(q)$ are some analogues for Fock spaces of Kazhdan–Lusztig R-polynomials $R_{x,y}(q)$ for Hecke algebras. The classical computation of $R'_{x,y}(1)$ was made in [GJ], in relation with the Kazhdan–Lusztig conjecture for multiplicities of composition factors of Verma modules.

Notation 1.1. Let $\mathbb{N}$ (respectively $\mathbb{N}^*$) denote the set of non-negative (respectively positive) integers, and for $a, b \in \mathbb{R}$ denote by $[\![a; b]\!]$ the discrete interval $[a, b] \cap \mathbb{Z}$. Throughout this article, we fix three integers $n, l, m \geqslant 1$. Let Π be the set of partitions of any integer and Π_m^l be the set of l-multi-partitions of m. The Coxeter group of type A_{r-1} (with $r \in \mathbb{N}^*$) is the symmetric group $\mathfrak{S}_r = \langle \sigma_i = (i, i+1) \mid 1 \leqslant i \leqslant r-1 \rangle$. Let ℓ be the length function on $\mathfrak{S}_r$ and ω be the unique element of maximal length in $\mathfrak{S}_r$.

Part A: Statement of results

2. Statement of results

2.1. The Jantzen sum formula

Definition 2.1. [AK,BM] Let R be a principal ideal domain. Let v be an invertible element of R and $u_1, \dots, u_l \in R$. The *Ariki–Koike algebra*, denoted by