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# The limiting distributions of large heavy Wigner and arbitrary random matrices



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## ABSTRACT

A heavy Wigner matrix  $X_N$  is defined similarly to a classical Wigner one. It is Hermitian, with independent sub-diagonal entries. The diagonal entries and the non-diagonal entries are identically distributed. Nevertheless, the moments of the entries of  $\sqrt{N}X_N$  tend to infinity with  $N$ , as for matrices with truncated heavy tailed entries or adjacency matrices of sparse Erdős–Rényi graphs. Consider a family  $\mathbf{X}_N$  of independent heavy Wigner matrices and an independent family  $\mathbf{Y}_N$  of arbitrary random matrices with a bound condition and converging in  $*$ -distribution in the sense of free probability. We characterize the possible limiting joint  $*$ -distributions of  $(\mathbf{X}_N, \mathbf{Y}_N)$ , giving explicit formulas for joint  $*$ -moments. We find that they depend on more than the  $*$ -distribution of  $\mathbf{Y}_N$  and that in general  $\mathbf{X}_N$  and  $\mathbf{Y}_N$  are not asymptotically  $*$ -free. We use the traffic distributions and the associated notion of independence [21] to encode the information on  $\mathbf{Y}_N$  and describe the limiting  $*$ -distribution of  $(\mathbf{X}_N, \mathbf{Y}_N)$ . We develop this approach for related models and give recurrence relations for the limiting  $*$ -distribution of heavy Wigner and independent diagonal matrices.

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**1. Introduction**

*1.1. Motivations*

*Notations.* While considering a matrix  $A_N$ , we implicitly mean a sequence  $(A_N)_{N \geq 1}$ , the matrix  $A_N$  being of size  $N$ . We often consider families of matrices, denoted by bold characters e.g.  $\mathbf{A}_N = (A_1, \dots, A_p)$  or  $\mathbf{Y}_N = (Y_j)_{j \in J}$ . As well, we will often consider families of indeterminates (non-commutative formal variables), i.e. families of symbols e.g.  $\mathbf{a} = (a_1, \dots, a_p)$  or  $\mathbf{y} = (y_j)_{j \in J}$ . By  $\mathbf{a}^*$  and  $\mathbf{y}^*$  we mean families of indeterminates  $(a_1^*, \dots, a_p^*), (y_j^*)_{j \in J}$ .

Let  $\mathbf{A}_N$  be a family of random complex matrices, whose entries have finite moments of all orders (for each matrix  $A$  of  $\mathbf{A}_N$  one has  $\mathbb{E}[|A(i, j)|^K] < \infty$  for any  $i, j, K \geq 1$ ). Following random matrix and free probability terminology, we call (mean) **\*-distribution** of  $\mathbf{A}_N$  the map

$$\Phi_{\mathbf{A}_N} : P \mapsto \mathbb{E} \left[ \frac{1}{N} \text{Tr} [P(\mathbf{A}_N)] \right],$$

defined on the space of non-commutative \*-polynomials, i.e. finite complex linear combinations of words in indeterminates  $\mathbf{a}$  and  $\mathbf{a}^*$ . When it exists, we call limiting

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