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The limiting distributions of large heavy Wigner and arbitrary random matrices

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A B S T R A C T

A heavy Wigner matrix X_N is defined similarly to a classical Wigner one. It is Hermitian, with independent sub-diagonal entries. The diagonal entries and the non-diagonal entries are identically distributed. Nevertheless, the moments of the entries of $\sqrt{N}X_N$ tend to infinity with N, as for matrices with truncated heavy tailed entries or adjacency matrices of sparse Erdös–Rényi graphs. Consider a family $\mathbf{X}_N$ of independent heavy Wigner matrices and an independent family $\mathbf{Y}_N$ of arbitrary random matrices with a bound condition and converging in *-distribution in the sense of free probability. We characterize the possible limiting joint *-distributions of $(\mathbf{X}_N, \mathbf{Y}_N)$, giving explicit formulas for joint *-moments. We find that they depend on more than the *-distribution of $\mathbf{Y}_N$ and that in general $\mathbf{X}_N$ and $\mathbf{Y}_N$ are not asymptotically *-free. We use the traffic distributions and the associated notion of independence [21] to encode the information on $\mathbf{Y}_N$ and describe the limiting *-distribution of $(\mathbf{X}_N, \mathbf{Y}_N)$. We develop this approach for related models and give recurrence relations for the limiting *-distribution of heavy Wigner and independent diagonal matrices.

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Contents

1. Introduction

1.1. Motivations

Notations. While considering a matrix A_N, we implicitly mean a sequence $(A_N)_{N\geqslant 1}$, the matrix A_N being of size N. We often consider families of matrices, denoted by bold characters e.g. $\mathbf{A}_N = (A_1, \ldots, A_p)$ or $\mathbf{Y}_N = (Y_j)_{j\in J}$. As well, we will often consider families of indeterminates (non-commutative formal variables), i.e. families of symbols e.g. $\mathbf{a} = (a_1, \ldots, a_p)$ or $\mathbf{y} = (y_j)_{j\in J}$. By $\mathbf{a}^*$ and $\mathbf{y}^*$ we mean families of indeterminates $(a_1^*, \ldots, a_p^*)$, $(y_j^*)_{j\in J}$.

Let $\mathbf{A}_N$ be a family of random complex matrices, whose entries have finite moments of all orders (for each matrix A of $\mathbf{A}_N$ one has $\mathbb{E}[\,|A(i,j)|^K] < \infty$ for any $i, j, K \geqslant 1$). Following random matrix and free probability terminology, we call (mean) ***-distribution** of $\mathbf{A}_N$ the map

$$\Phi_{\mathbf{A}_N} : P \mapsto \mathbb{E}\Big[\frac{1}{N}\mathrm{Tr}\big[P(\mathbf{A}_N)\big]\Big],$$

defined on the space of non-commutative *-polynomials, i.e. finite complex linear combinations of words in indeterminates $\mathbf{a}$ and $\mathbf{a}^*$. When it exists, we call limiting

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