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The Vlasov–Poisson–Boltzmann system for the whole range of cutoff soft potentials

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ABSTRACT

The dynamics of dilute electrons can be modeled by the fundamental one-species Vlasov–Poisson–Boltzmann system which describes mutual interactions of the electrons through collisions in the self-consistent electrostatic field. For cutoff intermolecular interactions, although there is some progress on the construction of global smooth solutions to its Cauchy problem near Maxwellians recently, the problem for the case of very soft potentials remains unsolved. By introducing a new time–velocity weighted energy method and based on some new optimal temporal decay estimates on the solution itself and some of its derivatives with respect to both the spatial and the velocity variables, it is shown in this manuscript that the Cauchy problem of the one-species Vlasov–Poisson–Boltzmann system for all cutoff soft potentials does exist a unique global smooth solution for general initial perturbation which is unnecessary to satisfy the neutral condition imposed in [13] for the case of cutoff moderately soft potentials but is assumed to be small in certain weighted Sobolev spaces. Our approach applies also to the case of cutoff hard potentials and thus provides a satisfactory global well-posedness theory to the one-species Vlasov–Poisson–Boltzmann system near

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Maxwellians for the whole range of cutoff intermolecular interactions in the perturbative framework.

Contents

1. Introduction

The dynamics of dilute electrons can be modeled by the fundamental one-species Vlasov–Poisson–Boltzmann system (called VPB system in the sequel for simplicity) which describes mutual interactions of the electrons through collisions in the self-consistent electrostatic field

$$\partial_t f + \xi \cdot \nabla_x f + \nabla_x \phi \cdot \nabla_\xi f = Q(f, f), \tag{1.1}$$

$$\Delta_x \phi(t,x) = \int_{\mathbb{R}^3} f(t,x,\xi) d\xi - n_b(x), \qquad \lim_{|x|\to+\infty} \phi(t,x) = 0. \tag{1.2}$$

Here the unknown $f = f(t,x,\xi) \geq 0$ is the density distribution function of electrons located at $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ with velocity $\xi = (\xi_1, \xi_2, \xi_3) \in \mathbb{R}^3$ at time $t \geq 0$. The potential function $\phi = \phi(t,x)$ generating the self-consistent electrostatic field $\nabla_x \phi$ in (1.1) is coupled with $f(t,x,\xi)$ through the Poisson equation (1.2) where $n_b(x) > 0$ is the background charge which is assumed to be a positive constant in the rest of this manuscript denoting that the background charge is spatially homogeneous and in such a case, we can set $n_b(x) = 1$ without loss of generality. The bilinear collision operator $Q(f,g)$ is defined by, cf. [1,17,20]

$$Q(f,g) = \int_{\mathbb{R}^3} \int_{\mathbb{S}^2} |\xi - \xi_*|^\gamma q_0(\vartheta) \big\{ f(\xi'_*) g(\xi') - f(\xi_*) g(\xi) \big\} d\xi_* d\omega, \tag{1.3}$$

where (ξ, ξ_*) and (ξ', ξ'_*), denoting velocities of two particles before and after their collisions respectively, satisfy