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A moderate deviation principle for 2-D stochastic Navier–Stokes equations driven by multiplicative Lévy noises ☆

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A B S T R A C T

In this paper, we establish a moderate deviation principle for two-dimensional stochastic Navier–Stokes equations driven by multiplicative *Lévy* noises. The weak convergence method introduced by Budhiraja, Dupuis and Ganguly in [3] plays a key role.

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Poisson random measures
Tightness

1. Introduction

Consider the two-dimensional Navier–Stokes equation with Dirichlet boundary condition, which describes the time evolution of an incompressible fluid,

$$\frac{\partial u(t,x)}{\partial t} - \nu\Delta u(t,x) + (u(t,x)\cdot\nabla)u(t,x) + \nabla p(t,x) = f(t,x), \tag{1.1}$$

with the conditions

$$\begin{cases} (\nabla\cdot u)(t,x) = 0, & \text{for } \ x\in D, \quad t>0, \\ u(t,x) = 0, & \text{for } \ x\in\partial D, t\geq 0, \\ u(0,x) = u_0(x), & \text{for } \ x\in D, \end{cases} \tag{1.2}$$

where D is a bounded open domain of $\mathbb{R}^2$ with regular boundary ∂D, $u(t,x)\in\mathbb{R}^2$ denotes the velocity field at time t and position x, $\nu>0$ is the viscosity coefficient, $p(t,x)$ denotes the pressure field, f is a deterministic external force.

To formulate the Navier–Stokes equation, we introduce the following standard spaces: let

$$V = \left\{v\in H_0^1(D;\mathbb{R}^2) : \nabla\cdot v = 0, \text{ a.e. in } D\right\},$$

with the norm

$$\|v\|_V := \left(\int_D |\nabla v|^2 dx\right)^{\frac{1}{2}} = \|v\|,$$

and let H be the closure of V in the L^2-norm

$$|v|_H := \left(\int_D |v|^2 dx\right)^{\frac{1}{2}} = |v|.$$

Define the operator A (Stokes operator) in H by the formula

$$Au := -\nu P_H\Delta u, \qquad \forall u\in H^2(D;\mathbb{R}^2)\cap V,$$

where the linear operator P_H (Helmhotz–Hodge projection) is the projection operator from $L^2(D;\mathbb{R}^2)$ to H, and define the nonlinear operator B by