Journal of Functional Analysis 271 (2016) 3554–3587

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Stabilization and controllability of first-order integro-differential hyperbolic equations

Jean-Michel Coron [a,1], Long Hu [b,a,*,2], Guillaume Olive [a,3]

[a] Sorbonne Universités, UPMC Univ Paris 06, UMR 7598, Laboratoire Jacques-Louis Lions, 4 place Jussieu, 75252 Paris cedex 05, France
[b] School of Mathematics, Shandong University, Jinan, Shandong 250100, China

A R T I C L E   I N F O

Article history:
Received 5 November 2015
Accepted 25 August 2016
Available online 12 September 2016
Communicated by F.-H. Lin

Keywords:
Integro-differential equation
Stabilization
Controllability
Fredholm backstepping transformation

A B S T R A C T

In the present article we study the stabilization of first-order linear integro-differential hyperbolic equations. For such equations we prove that the stabilization in finite time is equivalent to the exact controllability property. The proof relies on a Fredholm transformation that maps the original system into a finite-time stable target system. The controllability assumption is used to prove the invertibility of such a transformation. Finally, using the method of moments, we show in a particular case that the controllability is reduced to the criterion of Fattorini.

* Corresponding author.
E-mail addresses: coron@ann.jussieu.fr (J.-M. Coron), hu@ann.jussieu.fr (L. Hu), oliveg@ljll.math.upmc.fr (G. Olive).

[1] JMC was supported by the ERC advanced grant 266907 (CPDENL) of the 7th Research Framework Programme (FP7).
[2] LH was supported by the ERC advanced grant 266907 (CPDENL) of the 7th Research Framework Programme (FP7), the Young Scholars Program of Shandong University (No. 2016WLJH52) and the Natural Science Foundation of China (No. 11601284).
[3] GO was partially supported by the ERC advanced grant 266907 (CPDENL) of the 7th Research Framework Programme (FP7).

http://dx.doi.org/10.1016/j.jfa.2016.08.018
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1. Introduction and main results

The purpose of this article is the study of the stabilization and controllability properties of the equation

$$\begin{cases} u_t(t,x) - u_x(t,x) = \int_0^L g(x,y)u(t,y)\,dy, & t \in (0,T),\ x \in (0,L), \\ u(t,L) = U(t), & t \in (0,T), \\ u(0,x) = u^0(x), & x \in (0,L). \end{cases} \tag{1.1}$$

In (1.1), $T > 0$ is the time of control, $L > 0$ the length of the domain. u^0 is the initial data and $u(t,\cdot) : [0,L] \longrightarrow \mathbb{C}$ is the state at time $t \in [0,T]$, $g : (0,L) \times (0,L) \longrightarrow \mathbb{C}$ is a given function in $L^2((0,L) \times (0,L))$ and, finally, $U(t) \in \mathbb{C}$ is the boundary control at time $t \in (0,T)$.

The stabilization and controllability of (1.1) started in [13]. The authors proved that the equation

$$\begin{cases} u_t(t,x) - u_x(t,x) = \int_0^x g(x,y)u(t,y)\,dy + f(x)u(t,0), & t \in (0,T),\ x \in (0,L), \\ u(t,L) = U(t), & t \in (0,T), \\ u(0,x) = u^0(x), & x \in (0,L), \end{cases}$$

with g and f continuous, is always stabilizable in finite time. The proof uses the backstepping approach introduced and developed by M. Krstic and his co-workers (see, in particular, the pioneer articles [2,16,19] and the reference book [14]). This approach consists in mapping (1.1) into the following finite-time stable target system

$$\begin{cases} w_t(t,x) - w_x(t,x) = 0, & t \in (0,T),\ x \in (0,L), \\ w(t,L) = 0, & t \in (0,T), \\ w(0,x) = w^0(x), & x \in (0,L), \end{cases}$$

by means of the Volterra transformation of the second kind

$$u(t,x) = w(t,x) - \int_0^x k(x,y)w(t,y)dy, \tag{1.2}$$

where the kernel k has to satisfy some PDE in the triangle $0 \leq y \leq x \leq L$ with appropriate boundary conditions, the so-called kernel equation. Let us emphasize that the strength of this method is that the Volterra transformation (1.2) is always invertible