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Nonlinear stochastic partial differential equations with singular diffusivity and gradient Stratonovich noise

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A B S T R A C T

We study existence and uniqueness of a variational solution in terms of stochastic variational inequalities (SVI) to stochastic nonlinear diffusion equations with a highly singular diffusivity term and multiplicative Stratonovich gradient-type noise. We derive a commutator relation for the unbounded noise coefficients in terms of a geometric Killing vector condition. The drift term is given by the total variation flow, respectively, by a singular p-Laplace-type operator. We impose nonlinear zero Neumann boundary conditions and precisely investigate their connection with the coefficient fields of the noise. This solves an open problem posed in Barbu et al. (2013) [7] and Barbu and Röckner (2015) [10].

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1. Introduction

We consider existence and uniqueness of solutions to the following (multi-valued) nonlinear Stratonovich stochastic diffusion equation in $L^2(\mathcal{O})$,

$$\begin{cases} dX_t \in \operatorname{div}\left[\operatorname{sgn}\left(\nabla X_t\right)\right] dt + \displaystyle\sum_{i=1}^{N} \langle b_i, \nabla X_t\rangle \circ d\beta_t^i, & \text{in } (0,T)\times\mathcal{O}, \\ X_0 = x, & \text{in } \mathcal{O}, \\ \dfrac{\partial X_t}{\partial \nu} = 0, & \text{on } (0,T)\times\partial\mathcal{O}, \end{cases} \tag{1.1}$$

where $\mathcal{O}$ is an open, bounded domain in $\mathbb{R}^d$, $d \geq 2$, with (sufficiently) smooth boundary such that $\mathcal{O}$ or $\partial\mathcal{O}$ is convex. Here, for $N \geq 1$, $b_i : \overline{\mathcal{O}} \to \mathbb{R}^d$, $1 \leq i \leq N$ are "coefficient fields" and $\beta = (\beta^1, \ldots, \beta^N)$ denotes an N-dimensional Brownian motion on a filtered (normal) probability space $\left(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P}\right)$. The initial datum is chosen as $x \in L^2(\mathcal{O})$, or, more generally, as $x \in L^2(\Omega, \mathcal{F}_0, \mathbb{P}; L^2(\mathcal{O}))$. Here, ν denotes the outer unit normal on $\partial\mathcal{O}$. The multi-valued graph $\xi \mapsto \operatorname{sgn}(\xi)$ from $\mathbb{R}^d$ into $2^{\mathbb{R}^d}$ is defined by

$$\operatorname{sgn}(\xi) := \begin{cases} \dfrac{\xi}{|\xi|}, & , \text{ if } \xi \neq 0, \\ \left\{\zeta \in \mathbb{R}^d \,|\, |\zeta| \leq 1\right\} & , \text{ if } \xi = 0, \end{cases}$$

for all $\xi \in \mathbb{R}^d$. Because of the multi-valued diffusivity term, the equation becomes formally a *stochastic evolution inclusion*, as have been studied e.g. in [38,32,31]. We denote by $|\cdot|$ the Euclidean norm of $\mathbb{R}^d$, and by $\langle\cdot,\cdot\rangle$ the Euclidean scalar product of $\mathbb{R}^d$.

Set

$$\mathbf{b} := \begin{pmatrix} b_1 \\ \vdots \\ b_N \end{pmatrix} : \overline{\mathcal{O}} \to \mathbb{R}^{N\times d}, \tag{1.2}$$

and denote by $\mathbf{b}^*$ its transpose. We have that equation (1.1) is formally equivalent to the Itô stochastic partial differential equation,

$$\begin{cases} dX_t \in \operatorname{div}\left[\operatorname{sgn}\left(\nabla X_t\right)\right] dt + \dfrac{1}{2}\operatorname{div}[\mathbf{b}^*\mathbf{b}\nabla X_t]\, dt + \langle \mathbf{b}\nabla X_t, d\beta_t\rangle, & \text{in } (0,T)\times\mathcal{O}, \\ X_0 = x, & \text{in } \mathcal{O}, \\ \dfrac{\partial X_t}{\partial \nu} = 0, & \text{on } (0,T)\times\partial\mathcal{O}. \end{cases} \tag{1.3}$$

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