Journal of Functional Analysis 271 (2016) 1341–1375

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Uniform time of existence for the alpha Euler equations

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A R T I C L E I N F O

Article history:
Received 26 July 2015
Accepted 9 June 2016
Available online 17 June 2016
Communicated by C. De Lellis

Keywords:
Non-Newtonian fluids
Well-posedness for PDEs
Singular limit
Conormal spaces
Euler equation
Navier boundary conditions

A B S T R A C T

We consider the α-Euler equations on a bounded three-dimensional domain with frictionless Navier boundary conditions. Our main result is the existence of a strong solution on a positive time interval, uniform in α, for α sufficiently small. Combined with the convergence result in [6], this implies convergence of solutions of the α-Euler equations to solutions of the incompressible Euler equations when $\alpha \to 0$. In addition, we obtain a new result on local existence of strong solutions for the incompressible Euler equations on bounded three-dimensional domains. The proofs are based on new *a priori* estimates in conormal spaces.

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http://dx.doi.org/10.1016/j.jfa.2016.06.006

1. Introduction

The α-Euler equations, $\alpha > 0$, are a system of equations given by:

$$\partial_t(u - \alpha\Delta u) + u \cdot \nabla(u - \alpha\Delta u) + \sum_j (u - \alpha\Delta u)_j \nabla u_j = -\nabla p, \qquad \operatorname{div} u = 0, \quad (1)$$

where $u = (u_1, u_2, u_3)$ is the velocity and p is the scalar pressure.

These equations arise as the zero-viscosity case of the second grade fluids, a model of non-Newtonian fluids introduced in [7] as one among a hierarchy of models of viscoelastic fluids called fluids of differential type. The α-Euler equations are also used as a sub-grid scale model in turbulence and have been found to possess deep geometric significance, see [13].

Note that, if we formally set $\alpha = 0$ in (1), then we obtain the incompressible Euler equations:

$$\partial_t u + u \cdot \nabla u = -\nabla p, \qquad \operatorname{div} u = 0, \quad (2)$$

since $\sum_j u_j \nabla u_j$ is a gradient and can be absorbed by the pressure.

Existence of smooth solutions for system (1) has been established locally in time, in several contexts, see [4,13,17] and [5]. Global existence, however, is an open problem, a situation which parallels the outstanding open problem of existence of smooth solutions for the 3-dimensional Euler equations (2). The main concern of the present work is the existence of smooth solutions of the α-Euler equations (1) up to a time which is uniform with respect to α.

This problem needs to be considered in several fluid domains. In the case of flow in all of $\mathbb{R}^3$, existence of a smooth solution, with smooth initial data, was established for a time at least as long as the time of existence for $3D$ Euler, see [10]. For flow in a smooth, bounded domain with no-slip boundary conditions ($u = 0$), the problem remains open. In this paper we consider flow in a smooth, bounded domain $\Omega \subset \mathbb{R}^3$, with frictionless Navier boundary conditions, *i.e.*

$$u \cdot n = 0, \quad [D(u)n]\big|_{tan} = 0 \quad \text{on } \partial\Omega, \quad (3)$$

where $D(u)$ is the deformation tensor defined by $D(u) = \frac{1}{2}\big((\nabla u) + (\nabla u)^t\big)$ and the subscript “tan” denotes the tangential part. Our main result is to show that, given a sufficiently smooth initial velocity u_0, there exists a solution of (1) satisfying (3), for a time which is independent of α.

This analogous question may be posed for the Navier–Stokes equations,

$$\partial_t u + u \cdot \nabla u = -\nabla p + \nu\Delta u, \qquad \operatorname{div} u = 0, \quad (4)$$

namely, existence of solutions for a time independent of viscosity ν. For this problem, in the case of the flow in full-space it is classical that, if the initial velocity is sufficiently