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Absolute continuity for commuting row contractions

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Absolutely continuous commuting row contractions admit a weak-$*$ continuous functional calculus. Building on recent work describing the first and second dual spaces of the closure of the polynomial multipliers on the Drury–Arveson space, we give a complete characterization of these commuting row contractions in measure theoretic terms. We also establish that completely non-unitary row contractions are necessarily absolutely continuous, in direct parallel with the case of a single contraction. Finally, we consider refinements of this question for row contractions that are annihilated by a given ideal.

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1. Introduction

A single contraction T acting on a Hilbert space $\mathcal{H}$ can be successfully analyzed by using the theory developed by Sz.-Nagy and Foias in their seminal work [28]. Briefly, one splits the contraction into a direct sum $T = T_{\text{cnu}} \oplus U$ where U is a unitary operator and T_{cnu} is *completely non-unitary* in the sense that it has no closed invariant subspace on which it restricts to be a unitary operator. The study of T then reduces to the separate examination of the two pieces T_{cnu} and U. The unitary part U is rather well understood by virtue of the classical spectral theorem, and thus one is left with the task of understanding the completely non-unitary part. Fortunately, the geometric structure of the minimal unitary dilation of completely non-unitary contractions is especially transparent (Proposition II.1.4 in [28]). It can be deduced thereof that the polynomial functional calculus associated to a completely non-unitary contraction T extends to a unital algebra homomorphism

$$\Phi_T : H^\infty(\mathbb{D}) \to B(\mathcal{H})$$

which is completely contractive and weak-$*$ continuous. This is the celebrated Sz.-Nagy–Foias $H^\infty(\mathbb{D})$ functional calculus. In fact, this functional calculus exists for every contraction whose minimal unitary dilation has a spectral measure which is absolutely continuous with respect to Lebesgue measure on the circle. This last observation rests on the complete characterization of the so-called Henkin measures on the circle [9,19,26]. The existence of the map Φ_T has had a tremendous impact on single operator theory, making significant appearances in great advances in the invariant subspace problem [5] and in the classification of C_0 contractions [3] for instance; and it remains of fundamental importance in current work. In particular, the functional calculus creates a bridge between single operator theory and function theory in the classical Hardy space $H^2(\mathbb{D})$.

A topic of modern interest is the simultaneous study of several operators, or multivariate operator theory. One particular aspect of it is concerned with commuting operators $T_1, \ldots, T_d$ acting on the same Hilbert space $\mathcal{H}$ with the property that the row operator $T = (T_1, \ldots, T_d)$, which maps the space $\mathcal{H}^{(d)} = \mathcal{H} \oplus \mathcal{H} \oplus \ldots \oplus \mathcal{H}$ into $\mathcal{H}$ in the natural way, is contractive. We then say that $T = (T_1, \ldots, T_d)$ is a *commuting row contraction*. These have been the target of intense research in recent years, spurred on by the work of Arveson [1] who showed that commuting row contractions are deeply linked with a space of holomorphic functions on the open unit ball, now called the *Drury–Arveson space*. In fact, there always exists a contractive functional calculus

$$\Phi_T : \mathcal{A}_d \to B(\mathcal{H})$$

where $\mathcal{A}_d$ is the closure of the polynomials in the multiplier norm, a norm which dominates the usual supremum norm over the ball. The resulting analogy with the single operator correspondence between contractions and function theory on the disc has proved quite fruitful, and this paper is a contribution to this program.