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# On the definition and the properties of the principal eigenvalue of some nonlocal operators

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## ABSTRACT

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In this article we study some spectral properties of the linear operator  $\mathcal{L}_\Omega + a$  defined on the space  $C(\bar{\Omega})$  by:

$$\mathcal{L}_\Omega[\varphi] + a\varphi := \int_{\Omega} K(x, y)\varphi(y) dy + a(x)\varphi(x)$$

where  $\Omega \subset \mathbb{R}^N$  is a domain, possibly unbounded,  $a$  is a continuous bounded function and  $K$  is a continuous, non-negative kernel satisfying an integrability condition.We focus our analysis on the properties of the generalised principal eigenvalue  $\lambda_p(\mathcal{L}_\Omega + a)$  defined by

$$\lambda_p(\mathcal{L}_\Omega + a) := \sup\{\lambda \in \mathbb{R} \mid \exists \varphi \in C(\bar{\Omega}), \varphi > 0,$$

such that  $\mathcal{L}_\Omega[\varphi] + a\varphi + \lambda\varphi \leq 0$  in  $\Omega$ .We establish some new properties of this generalised principal eigenvalue  $\lambda_p$ . Namely, we prove the equivalence of different definitions of the principal eigenvalue. We also study the behaviour of  $\lambda_p(\mathcal{L}_\Omega + a)$  with respect to some scaling of  $K$ .

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For kernels  $K$  of the type,  $K(x, y) = J(x - y)$  with  $J$  a compactly supported probability density, we also establish some asymptotic properties of  $\lambda_p(\mathcal{L}_{\sigma, m, \Omega} - \frac{1}{\sigma^m} + a)$  where  $\mathcal{L}_{\sigma, m, \Omega}$  is defined by  $\mathcal{L}_{\sigma, 2, \Omega}[\varphi] := \frac{1}{\sigma^{2+N}} \int_{\Omega} J\left(\frac{x-y}{\sigma}\right) \varphi(y) dy$ .

In particular, we prove that

$$\lim_{\sigma \rightarrow 0} \lambda_p \left( \mathcal{L}_{\sigma, 2, \Omega} - \frac{1}{\sigma^2} + a \right) = \lambda_1 \left( \frac{D_2(J)}{2N} \Delta + a \right),$$

where  $D_2(J) := \int_{\mathbb{R}^N} J(z)|z|^2 dz$  and  $\lambda_1$  denotes the Dirichlet principal eigenvalue of the elliptic operator. In addition, we obtain some convergence results for the corresponding eigenfunction  $\varphi_{p, \sigma}$ .

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## 1. Introduction

The principal eigenvalue of an operator is a fundamental notion in modern analysis. In particular, this notion is widely used in PDE's literature and is at the source of many profound results especially in the study of elliptic semi-linear problems. For example, the principal eigenvalue is used to characterise the stability of equilibrium of a reaction-diffusion equation enabling the definition of persistence criteria [18–20,5,33,44,53]. It is also an important tool in the characterisation of maximum principle properties satisfied by elliptic operators [12,8] and to describe continuous semi-groups that preserve an or-

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