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Integral operators mapping into the space of bounded analytic functions ☆

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ABSTRACT

We address the problem of studying the boundedness, compactness and weak compactness of the integral operators $T_g(f)(z) = \int_0^z f(\zeta)g'(\zeta)\,d\zeta$ acting from a Banach space X into H^∞. We obtain a collection of general results which are appropriately applied and mixed with specific techniques in order to solve the posed questions to particular choices of X.

Contents

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1. Introduction

Let $\mathcal{H}(\mathbb{D})$ denote the space of analytic functions in the unit disc $\mathbb{D} = \{z : |z| < 1\}$, and let $\mathbb{T}$ stand for the boundary of $\mathbb{D}$. The boundedness and compactness of the integral operators

$$T_g(f)(z) = \int_0^z f(\zeta)g'(\zeta)\, d\zeta, \quad g \in \mathcal{H}(\mathbb{D}),$$

have been extensively studied on different spaces of analytic functions since the seminal works by Aleman, Cima, Siskakis, and the third author of this paper [1,2,25]. However, it seems that little is known about T_g mapping a Banach space $X \subset \mathcal{H}(\mathbb{D})$ into the space H^∞ of bounded analytic functions in $\mathbb{D}$. As for this question, a good number of results were proved in [4] when $X = H^\infty$. Moreover, a classical result attributed to Privalov [27] shows that the Volterra operator $V(f)(z) = \int_0^z f(\zeta)\, d\zeta$ is bounded from the Hardy space H^1 to the disc algebra A, the space of those $f \in \mathcal{H}(\mathbb{D})$ which are continuous on $\overline{\mathbb{D}}$. The main objective of this paper is to study the boundedness, compactness and weak compactness of T_g acting from X into H^∞. To begin with, we will work in a wide framework providing abstract approaches to these questions. Later on, these general results will be applied to particular choices of X.

In order to explain the first of our methods some definitions are needed. Let $\{\beta_n\}_{n=0}^\infty$ be a sequence of positive numbers such that $\lim_{n\to\infty} \sqrt[n]{\beta_n} = 1$, and let $H(\beta)$ be the Hilbert space of analytic functions in $\mathbb{D}$ induced by the $H(\beta)$-pairing

$$\langle f, g\rangle_{H(\beta)} = \lim_{r\to 1^-} \sum_{n=0}^{\infty} \widehat{f}(n)\overline{\widehat{g}(n)}\beta_n r^n,$$

where $f(z) = \sum_{n=0}^\infty \widehat{f}(n)z^n$ and $g(z) = \sum_{n=0}^\infty \widehat{g}(n)z^n$. For $f \in \mathcal{H}(\mathbb{D})$ and $0 < r < 1$, define $f_r(z) = f(rz)$ for all $z \in \mathbb{D}$. Throughout the paper a Banach space $X \subset \mathcal{H}(\mathbb{D})$ is called admissible if it satisfies the following conditions: