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Global well-posedness of helicoidal Euler equations

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ABSTRACT

This paper deals with the global existence and uniqueness results for the three-dimensional incompressible Euler equations with a particular structure for initial data lying in critical spaces. In this case the BKM criterion is not known.

1. Introduction

The purpose of this paper is to investigate the global well-posedness of the following three-dimensional incompressible Euler system in the whole space with helicoidal initial data. This system is described as follows:

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$$(\mathrm{E})\qquad \begin{cases} \partial_t u + (u\cdot\nabla)u + \nabla\Pi = 0, & (t,x)\in\mathbb{R}^+\times\mathbb{R}^3,\\ \operatorname{div} u = 0, \\ u_{|t=0} = u^0. \end{cases}$$

Here, the vector field $u = (u_1, u_2, u_3)$ is the velocity of the fluid and Π is a scalar pressure function. The operator $u.\nabla$ is given explicitly by $u.\nabla = \sum_{j=1}^{3} u_j\partial_j$ and the incompressibility of the fluid is expressed via the second equation of the system $\operatorname{div} u = \sum_{j=1}^{3}\partial_j u_j = 0$.

The question of local or global existence and uniqueness of solutions to the system (E) is one of the most important problems in fluid mechanics. Existence and uniqueness theories of (2 or 3 dimensional) Euler equations have been studied by many mathematicians and physicists. W. Wolibner [25] started the subject in Hölder spaces, D. Ebin [11], J. Bourguignon [3], R. Temam [20], T. Kato and G. Ponce [14] worked out this subject in Sobolev spaces. Much of the studies on the Euler equations of an ideal incompressible fluid in Besov spaces have been done by M. Vishik ([22–24]), D. Chae [7] and C. Park and J. Park [15].

The question of global existence (even for smooth initial data) is still open and continues to be one of the most challenging problems in nonlinear PDEs. The degree of difficulty depends strongly on the dimensions (2 or 3) and the regularity of the initial data. In this context, the vorticity plays a fundamental role. In fact, the well-known BKM criterion [4] ensures that the development of finite time singularities for Kato's solutions is related to the blowup of the L^∞ norm of the vorticity near the maximal time of existence. In 2-D, the vorticity satisfies a transport equation

$$\partial_t\omega + (u\cdot\nabla)\omega = 0.$$

In space dimension three, the vorticity satisfies the equation

$$\partial_t\omega + (u\cdot\nabla)\omega = (\omega\cdot\nabla)u \tag{1}$$

and the main difficulty for establishing global regularity is to understand how the vortex-stretching term $(\omega\cdot\nabla)u$ affects the dynamic of the fluid. While the question of global existence for 3-D Euler system is widely open, some positive results are available for the 3-D flows with some geometry constraints as the so-called axisymmetric flows without swirl. We say that a vector field u is axisymmetric if it has the form:

$$u(x,t) = u_r(r,z,t)e_r + u_z(r,z,t)e_z, \quad x = (x_1,x_2,z), \quad r = (x_1^2+x_2^2)^{\frac{1}{2}},$$

where $\big(e_r, e_\theta, e_z\big)$ is the cylindrical basis of $\mathbb{R}^3$ and the components u_r and u_z do not depend on the angular variable. The main feature of axisymmetric flows arises in the vorticity which takes the form