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Toeplitz algebras associated to endomorphisms of Ore semigroups

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ABSTRACT

In this paper, we consider the Toeplitz algebra associated to actions of Ore semigroups on C^*-algebras. In particular, we consider injective and surjective actions of such semigroups. We use the theory of groupoid dynamical systems to represent the Toeplitz algebra as a groupoid crossed product. We also discuss the K-theory of the Toeplitz algebra in some examples. For instance, we show that for the semigroup of positive matrices, the K-theory of the associated Toeplitz algebra vanishes.

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1. Introduction

Group C^*-algebras and the associated crossed products have always been at the centre of interest in the theory of C^*-algebras. In the nineties, Gerard J. Murphy in a series of papers studied C^*-algebras associated to semigroups and semigroup actions. The reader is referred to [18,17] and the references therein for the work of Murphy on semigroup C^*-algebras. The study of semigroup C^*-algebras is revived again in the last few years with the work [2] of Cuntz on the C^*-algebras associated to the "ax+b"-semigroup. We refer to [11,12] for recent developments on semigroup C^*-algebras. Even though we can now say that the literature on semigroup C^*-algebras and semigroup crossed products is vast, the literature on C^*-algebras associated to topological semigroups is scant in comparison to that on discrete semigroups.

The paper [14] by Muhly and Renault on Wiener–Hopf C^*-algebras associated to cones in $\mathbb{R}^n$ can be considered as the first systematic attempt in the direction of topological semigroups. Renault and Muhly made extensive use of groupoid techniques to explore the Wiener–Hopf algebras associated to polyhedral cones and self-dual cones. Later the works of Nica [19,20], Hilgert and Neeb [6,5] extended the analysis initiated in [14] to other semigroups, in particular by Hilgert and Neeb, to subsemigroups of Lie groups. In [21], based on the techniques used in [6], the Wiener–Hopf C^*-algebras associated to Ore semigroups are studied and a groupoid picture is obtained for them. Nevertheless, to the author's knowledge, the only reference in the literature where Wiener–Hopf or Toeplitz type C^*-algebra associated to actions of topological semigroups is considered is [7]. The action of the semigroup of the positive real line on a C^*-algebra is considered by Khoshkam and Skandalis in [7]. In fact, the present paper is an outgrowth of the author's effort to understand [7], the author's desire to place [7] within the framework of groupoids and to extend some of the results in [7] to possibly more general semigroups. This article owes its existence to [7]. In fact, the title of this paper is inspired by that of [7].

A brief outline of the problem considered in this paper is given below. First let us recall the construction of the reduced crossed product. Let G be a locally compact group, let A be a C^*-algebra and let $\alpha : G \to Aut(A)$ be a strongly continuous action. Consider the Hilbert A-module $E := A \otimes L^2(G) = L^2(G, A)$. Let $\mathcal{L}_A(E)$ be the C^*-algebra of adjointable operators on E. Define $\widetilde{\pi} : A \to \mathcal{L}_A(E)$ and $\widetilde{U} : G \to \mathcal{L}_A(E)$ by the formulas:

$$\widetilde{\pi}(x)(\xi)(h) := \alpha_h^{-1}(x)\xi(h)$$
$$\widetilde{U}_g(\xi)(h) := \xi(g^{-1}h)$$