Journal of Functional Analysis 270 (2016) 4087–4116

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Life-span of classical solutions to two dimensional fully nonlinear wave equations

Wei Han [a,*], Zhen Jin [b]

[a] Department of Mathematics, North University of China, Taiyuan, Shanxi 030051, PR China
[b] Complex Systems Research Center, Shanxi University, Taiyuan, Shanxi 030006, PR China

ARTICLE INFO

Article history:
Received 19 June 2014
Accepted 28 March 2016
Available online 7 April 2016
Communicated by Stefaan Vaes

Keywords:
Fully nonlinear wave equations
Life-span
Cauchy problem
Strichartz estimates

ABSTRACT

In two-space-dimensional case, we get the lower bound of the life-span of classical solutions to the Cauchy problem with small initial data for fully nonlinear wave equations of the form $\Box u = F(u, Du, D_x Du)$ in which $F(\widehat{\lambda}) = O(|\widehat{\lambda}|^3)$ in a neighbourhood of $\widehat{\lambda} = 0$, and $\partial_u^3 F(0,0,0) = 0$. For the purpose, some refined estimates including Strichartz estimates are needed in the paper.

1. Introduction and main results

Consider the Cauchy problem for fully nonlinear wave equations

$$\begin{cases} \Box u = F(u, Du, D_x Du), \quad x \in R^2, \quad t > 0, \\ t = 0: \; u = \varepsilon\varphi(x), \quad u_t = \varepsilon\psi(x), \quad x \in R^2, \end{cases} \tag{1.1}$$

where

* Corresponding author.
E-mail addresses: sh_hanweiwei1@126.com (W. Han), jinzhn@263.net (Z. Jin).

http://dx.doi.org/10.1016/j.jfa.2016.03.016

$$x = (x_1, x_2), \quad \text{and} \quad \square = \frac{\partial^2}{\partial t^2} - \sum_{i=1}^{2} \frac{\partial^2}{\partial x_i^2} \tag{1.2}$$

is the wave operator,

$$D = \left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2} \right), \quad D_x = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2} \right), \tag{1.3}$$

$$\varphi,\ \psi \in C_0^\infty(R^2) \tag{1.4}$$

and $\varepsilon > 0$ is a small parameter.

Let

$$\widehat{\lambda} = (\lambda; (\lambda_i), i = 0,\ 1, 2, (\lambda_{ij}), i, j = 0, 1, 2,\ i + j \geq 1)\,. \tag{1.5}$$

Suppose that in a neighbourhood of $\widehat{\lambda} = 0$, say for $|\widehat{\lambda}| \leq 1$, the nonlinear term $F = F(\widehat{\lambda})$ in (1.1) is a sufficiently smooth function satisfying

$$F(\widehat{\lambda}) = O(|\widehat{\lambda}|^3). \tag{1.6}$$

Our aim is to study the life-span of classical solution to (1.1) for $n = 2$ and cubic nonlinearity. By definition, the life-span $\widetilde{T}(\varepsilon) = \sup \tau$ for all $\tau > 0$ such that there exists a classical solution to (1.1) on $0 \leq t \leq \tau$. For the general nonlinear wave equations with small initial data in the case $n = 2$ and cubic nonlinearity, we have the following lower bounds of the life-span of the solutions:

(i) In the general case

$$\widetilde{T}(\varepsilon) \geq b\varepsilon^{-6}; \tag{1.7}$$

(ii) If

$$\partial_u^3 F(0, 0, 0) = \partial_u^4 F(0, 0, 0) = 0, \tag{1.8}$$

then

$$\widetilde{T}(\varepsilon) \geq \exp(a\varepsilon^{-2}), \tag{1.9}$$

where a, b are positive constants independent of ε. For these results, T.T. Li and Y. Zhou in [11] obtained in a direct and simple manner the above lower bounds (1.7) and (1.9) for the general case and for the special case $\partial_u^\beta F(0, 0, 0) = 0 (\beta = 3, 4)$ in the case of $n = 2$ and the cubic nonlinearity. These estimates are both sharp, see Y. Zhou [18,17], Y. Zhou and W. Han [19]. For other results on the life span for fully nonlinear wave equations in two space dimensions, one can see L. Hörmander [2], M. Kovalyov [6], H. Lindblad [14],