



ELSEVIER

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa



Global uniqueness theorems for linear and nonlinear waves



Spyros Alexakis^{a,*}, Arick Shao^b

^a Department of Mathematics, University of Toronto, 40 St George Street
Rm 6290, Toronto, ON M5S 2E4, Canada

^b Department of Mathematics, South Kensington Campus, Imperial College
London, London SW7 2AZ, United Kingdom

ARTICLE INFO

Article history:

Received 8 January 2015

Accepted 17 August 2015

Available online 1 October 2015

Communicated by S. Brendle

Keywords:

Wave equations

Radiation field

Unique continuation

Nonlinear waves

ABSTRACT

We prove a unique continuation from infinity theorem for regular waves of the form $[\square + \mathcal{V}(t, x)]\phi = 0$. Under the assumption of no incoming and no outgoing radiation on specific halves of past and future null infinities, we show that the solution must vanish everywhere. The “no radiation” assumption is captured in a specific, *finite* rate of decay which in general depends on the L^∞ -profile of the potential \mathcal{V} . We show that the result is optimal in many regards. These results are then extended to certain power-law type nonlinear wave equations, where the order of decay one must assume is independent of the size of the nonlinear term. These results are obtained using a new family of global Carleman-type estimates on the exterior of a null cone. A companion paper to this one explores further applications of these new estimates to such nonlinear waves.

© 2015 Elsevier Inc. All rights reserved.

* Corresponding author.

E-mail addresses: alexakis@math.utoronto.ca (S. Alexakis), c.shao@imperial.ac.uk (A. Shao).

Contents

1.	Introduction	3459
1.1.	The main results	3463
1.1.1.	Linear wave equations	3464
1.1.2.	Nonlinear wave equations	3467
1.2.	The main estimates	3469
2.	Global estimates	3471
2.1.	The preliminary estimate	3471
2.1.1.	Preliminaries	3473
2.1.2.	Proof of Proposition 2.5	3473
2.2.	The linear estimate	3476
2.2.1.	Proof of Theorem 2.12	3478
2.3.	The nonlinear estimate	3480
2.3.1.	Proof of Theorem 2.17	3481
3.	Proofs of the main results	3482
3.1.	Special domains	3483
3.1.1.	Basic properties	3484
3.1.2.	Boundary expansions	3485
3.2.	Boundary limits	3487
3.2.1.	Coarea formulas	3487
3.2.2.	The conformal inversion	3489
3.2.3.	The boundary limit lemmas	3490
3.3.	Proof of Theorem 1.1	3492
3.4.	Proofs of Theorems 1.6 and 1.7	3495
3.5.	Proof of Proposition 1.2	3497
3.5.1.	Construction of ψ	3497
	Acknowledgments	3498
	References	3498

1. Introduction

This paper presents certain global unique continuation results for linear and nonlinear wave equations. The motivating challenge is to investigate the extent to which globally regular waves can be reconstructed from the radiation they emit towards (suitable portions of) null infinity. We approach this in the sense of uniqueness: if a regular wave emits no radiation towards appropriate portions of null infinity, then it must vanish.

The belief that a lack of radiation emitted towards infinity should imply the triviality of the underlying solution has been implicit in the physics literature for many classical fields. For instance, in the case of linear Maxwell equations, early results in this direction go back at least to [25]. Moreover, in general relativity, the question whether non-radiating gravitational fields must be trivial (i.e., stationary) goes back at least to [22], in connection with the possibility of time-periodic solutions of Einstein's equations. The presumption that the answer must be affirmative under suitable assumptions underpins many of the central stipulations in the field; see for example the issue of the final state in [10].

We note that the reconstruction of *free* waves in the Minkowski space–time from their radiation fields (for smooth enough initial data) is classically known. For instance, this

Download English Version:

<https://daneshyari.com/en/article/4589704>

Download Persian Version:

<https://daneshyari.com/article/4589704>

[Daneshyari.com](https://daneshyari.com)