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Global uniqueness theorems for linear and nonlinear waves

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ABSTRACT

We prove a unique continuation from infinity theorem for regular waves of the form $[\Box + \mathcal{V}(t,x)]\phi = 0$. Under the assumption of no incoming and no outgoing radiation on specific halves of past and future null infinities, we show that the solution must vanish everywhere. The "no radiation" assumption is captured in a specific, *finite* rate of decay which in general depends on the L^∞-profile of the potential $\mathcal{V}$. We show that the result is optimal in many regards. These results are then extended to certain power-law type nonlinear wave equations, where the order of decay one must assume is independent of the size of the nonlinear term. These results are obtained using a new family of global Carleman-type estimates on the exterior of a null cone. A companion paper to this one explores further applications of these new estimates to such nonlinear waves.

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Contents

1. Introduction

This paper presents certain global unique continuation results for linear and nonlinear wave equations. The motivating challenge is to investigate the extent to which globally regular waves can be reconstructed from the radiation they emit towards (suitable portions of) null infinity. We approach this in the sense of uniqueness: if a regular wave emits no radiation towards appropriate portions of null infinity, then it must vanish.

The belief that a lack of radiation emitted towards infinity should imply the triviality of the underlying solution has been implicit in the physics literature for many classical fields. For instance, in the case of linear Maxwell equations, early results in this direction go back at least to [25]. Moreover, in general relativity, the question whether non-radiating gravitational fields must be trivial (i.e., stationary) goes back at least to [22], in connection with the possibility of time-periodic solutions of Einstein's equations. The presumption that the answer must be affirmative under suitable assumptions underpins many of the central stipulations in the field; see for example the issue of the final state in [10].

We note that the reconstruction of *free* waves in the Minkowski space–time from their radiation fields (for smooth enough initial data) is classically known. For instance, this