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Integration operators between Hardy spaces on the unit ball of $\mathbb{C}^n$

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We completely describe the boundedness of the Volterra type operator J_g between Hardy spaces in the unit ball of $\mathbb{C}^n$. The proof of the one dimensional case used tools, such as the strong factorization for Hardy spaces, that are not available in higher dimensions, and therefore other techniques must be used. In particular, a generalized version of the description of Hardy spaces in terms of the area function is needed.

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1. Introduction and main results

Let $\mathbb{B}_n$ be the open unit ball in $\mathbb{C}^n$. Denote by $H(\mathbb{B}_n)$ the space of all holomorphic functions in $\mathbb{B}_n$. For a function $g \in H(\mathbb{B}_n)$, define the operator

$$J_g f(z) = \int_0^1 f(tz) Rg(tz) \frac{dt}{t}, \qquad z \in \mathbb{B}_n \tag{1.1}$$

for f holomorphic in $\mathbb{B}_n$. Here Rg denotes the radial derivative of g, that is,

$$Rg(z) = \sum_{k=1}^{n} z_k \frac{\partial g}{\partial z_k}(z), \qquad z = (z_1, \dots, z_n) \in \mathbb{B}_n.$$

In the one dimensional case $n = 1$, the operator J_g was first considered in the setting of Hardy spaces by Pommerenke [32] related to the study of certain properties of $BMOA$ functions. We want to mention here that a closely related operator was introduced earlier by Calderón in [10]. After the pioneering works of Aleman, Siskakis and Cima [4,6,7] describing the boundedness and compactness of the operator J_g in Hardy and Bergman spaces, the mentioned operator became extremely popular, being studied in many spaces of analytic functions (see [4–7,14,29,30] for example). As far as we know, the generalization of the operator J_g acting on holomorphic functions in the unit ball of $\mathbb{C}^n$ (as defined here) was introduced by Z. Hu [20]. A fundamental property of the operator J_g, that follows from an easy calculation with (1.1), is the following basic formula involving the radial derivative R and the operator J_g:

$$R(J_g f)(z) = f(z)\, Rg(z), \quad z \in \mathbb{B}_n. \tag{1.2}$$

The boundedness and compactness of J_g has been extensively studied in many spaces of holomorphic functions in the unit ball (see [39] and [40] for the corresponding study on Bergman and Bloch type spaces). However, the case of the Hardy spaces on the unit ball, that is, the study of $J_g : H^p(\mathbb{B}_n) \to H^q(\mathbb{B}_n)$ (that, in my opinion, is the most important case, and is the setting where the operator J_g was originally studied) is missing, only the elementary case $q = p = 2$ (see [23]) and the case $p < q$ (see [9]) has been done before. Our goal is to fill this gap, and we completely describe the boundedness and compactness of $J_g : H^p(\mathbb{B}_n) \to H^q(\mathbb{B}_n)$ for all $0 < p, q < \infty$.

For $0 < p < \infty$, the Hardy space $H^p := H^p(\mathbb{B}_n)$ consists of those holomorphic functions f in $\mathbb{B}_n$ with

$$\|f\|_{H^p}^p = \sup_{0<r<1} \int_{\mathbb{S}_n} |f(r\zeta)|^p \, d\sigma(\zeta) < \infty,$$