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Continuous unital dilations of completely positive semigroups

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ABSTRACT

Since it began in the 1970s, the study of completely positive semigroups has included among its central topics the dilation of a completely positive semigroup to an endomorphism semigroup. Several authors have proved the existence of dilations, but have usually only achieved unital dilations of specific examples, while general theorems have tended to produce non-unital dilations. A unique approach due to Jean-Luc Sauvageot leads to a unital dilation in a general setting, but leaves unclear the continuity of the dilation semigroup. The major purpose of this paper, therefore, is to further develop Sauvageot's theory in order to prove the existence of continuous unital dilations.

The dilation is characterized by a combinatorial property akin to free independence. This property is implicit in some of Sauvageot's original calculations, but a secondary goal of this paper is to present it as its own object of study, which is done in Section 4.

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1. Introduction

The main objects of study in this paper are dilations of completely positive semigroups to endomorphism semigroups. A **completely positive semigroup**, or **cp-semigroup**, on a C*-algebra (resp. W*-algebra) A is a family $\{\phi_t\}_{t\geq 0}$ of (normal) completely positive linear maps $\phi_t: A \to A$ such that ϕ_0 is the identity map and $\phi_t \circ \phi_s = \phi_{t+s}$ for all $s, t \geq 0$. An **endomorphism semigroup**, or **e-semigroup**, is a cp-semigroup in which each ϕ_t is a $*$-homomorphism. In case A has a unit, a **cp$_0$-** or **e$_0$-semigroup** is one in which each ϕ_t is unital. Capital letters in the terms **CP-semigroup**, **E$_0$-semigroup**, etc. indicate that for each fixed $a \in A$, the map $t \mapsto \phi_t(a)$ is continuous in the norm (resp. σ-weak) topology on A.

A **dilation** of a cp-semigroup is a quadruple $(\mathfrak{A}, i, \mathbb{E}, \sigma)$ in which $\mathfrak{A}$ is a C*-algebra (resp. W*-algebra); $i: A \to \mathfrak{A}$ is an isometric $*$-homomorphism, hereafter termed an **embedding**; $\sigma = \{\sigma_t\}_{t\geq 0}$ is an e-semigroup on $\mathfrak{A}$; and $\mathbb{E} : \mathfrak{A} \to A$ is a completely positive contraction with the property that $\mathbb{E} \circ \sigma_t \circ i = \phi_t$ for all $t \geq 0$, corresponding to the following diagram.

$$\begin{CD} \mathfrak{A} @>{\sigma_t}>> \mathfrak{A} \\ @A{i}AA @VV{\mathbb{E}}V \\ A @>>{\phi_t}> A \end{CD}$$

Note in particular (from the case $t = 0$) that $\mathbb{E} \circ i$ is the identity map on A (we say $\mathbb{E}$ is a **retraction** with respect to i) and that $i \circ \mathbb{E}$ is a conditional expectation from $\mathfrak{A}$ onto the image of A. A **strong dilation** is one for which $\mathbb{E} \circ \sigma_t = \phi_t \circ \mathbb{E}$ for all $t \geq 0$; among the advantages of a strong dilation is that, for any $\{\phi_t\}$-invariant state ω on A, the state $\omega \circ \mathbb{E}$ on $\mathfrak{A}$ is $\{\sigma_t\}$-invariant.

We will always require that a dilation of a CP-, cp$_0$-, or CP$_0$-semigroup be an E-, e$_0$-, or E$_0$-semigroup. However, in the latter two cases we will not always assume that the embedding $i: A \to \mathfrak{A}$ maps the unit of A to the unit of $\mathfrak{A}$. If it does, we say the dilation is **unital**.

The study of completely positive semigroups and their dilations originated in the 1970s as a model for quantum-mechanical systems which are "open" in a thermodynamic sense [9]. The existence of dilations for uniformly continuous CP-semigroups on $B(H)$ was proved in [11], using results of [13,19,8] on the structure of the generator of such a semigroup. Special cases for non-uniformly-continuous CP-semigroups were addressed by [10,1,27,17], and others. After the theory of product systems was developed in [2], more general dilation results followed: The existence of an E$_0$-dilation for every CP$_0$-semigroup was proved for $B(H)$ in [4] and [24], for general C*-algebras in [5], and for von Neumann algebras with separable predual in [6] and [20]. All of the dilations constructed are non-unital, as all rely at some point on the embedding of $B(H)$ in $B(K)$ for Hilbert spaces $H \subseteq K$.

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