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Entropy dissipation estimates for the linear Boltzmann operator



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ABSTRACT

We prove a linear inequality between the entropy and entropy dissipation functionals for the linear Boltzmann operator (with a Maxwellian equilibrium background). This provides a positive answer to the analogue of Cercignani's conjecture for this linear collision operator. Our result covers the physically relevant case of hard-spheres interactions as well as Maxwellian kernels, both with and without a cut-off assumption. For Maxwellian kernels, the proof of the inequality is surprisingly simple and relies on a general estimate of the entropy of the gain operator due to [27,32]. For more general kernels, the proof relies on a comparison principle. Finally, we also show that in the grazing collision limit our results allow to recover known logarithmic Sobolev inequalities.

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1. Introduction

1.1. Setting of the problem and main result

The use of Lyapunov functionals is a well-known technique to study the asymptotic behavior of dynamical systems, and in the theory of the Boltzmann equation and related models it is now a classical tool. For the nonlinear, spatially homogeneous Boltzmann equation

$$\partial_t f = \mathcal{Q}(f, f), \quad f(0, v) = f_0(v), \quad v \in \mathbb{R}^d, t \geq 0, \tag{1.1}$$

posed for a function $f = f(t, v)$ depending on $t \geq 0$ and $v \in \mathbb{R}^d$, it is a well-known fact that $f(t, v)$ converges (as $t \rightarrow \infty$) towards the Maxwellian distribution M_f with same mass, momentum and energy as f_0 ,

$$M_f(v) = \frac{\varrho_f}{(2\pi E_f)^{d/2}} \exp\left(-\frac{|v - \mathbf{u}_f|^2}{2 E_f}\right), \quad v \in \mathbb{R}^d,$$

where

$$\left. \begin{aligned} \varrho_f &= \int_{\mathbb{R}^d} f(t, v) \, dv = \int_{\mathbb{R}^d} f_0(v) \, dv, \\ \varrho_f \mathbf{u}_f &= \int_{\mathbb{R}^d} f(t, v) v \, dv = \int_{\mathbb{R}^d} f_0(v) v \, dv, \\ d \varrho_f E_f &= \int_{\mathbb{R}^d} f(t, v) |v - \mathbf{u}_f|^2 \, dv = \int_{\mathbb{R}^d} f_0(v) |v - \mathbf{u}_f|^2 \, dv \end{aligned} \right\} \quad \text{for all } t \geq 0.$$

Notice that Eq. (1.1) conserves density, momentum and kinetic energy which explains why the above quantities ϱ_f , \mathbf{u}_f and E_f are constant in time. The *Shannon–Boltzmann relative entropy* of f with respect to the Maxwellian distribution M_f

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