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Expansion of the almost sure spectrum in the weak disorder regime

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ABSTRACT

The spectrum of random ergodic Schrödinger-type operators is almost surely a deterministic subset of the real line. The random operator can be considered as a perturbation of a periodic one. As soon as the disorder is switched on via a global coupling constant, the spectrum expands. We estimate how much the spectrum expands at its bottom for operators on $\ell^2(\mathbb{Z}^d)$.

Contents

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1. Introduction

Due to the self-averaging property of ergodic Schrödinger operators the resulting spectrum is almost surely a fixed subset of the real line. If a random operator is a perturbation of a periodic operator, it is of interest to know how the spectrum expands once we switch on the disorder via a global coupling constant. Apart from the genuine interest to identify the location of the spectrum, this is also of central importance when identifying energy regions corresponding to localized wavepackets.

Otherwise it may happen that one proves a Wegner estimate, a Lifschitz tail bound or a similar statement related to localization, and then later discovers that the considered energy regime belongs to the resolvent set.

In this paper we consider an ϵ-small random perturbation of a discrete translation-invariant operator and we study how the bottom of its spectrum behaves. By symmetry, similar estimates apply to the location of the maximum of the spectrum, in a weak disorder regime. To fix the ideas, let us introduce a prototypical example. Let $\mathcal{H} = \ell^2(\mathbb{Z}^d)$ and $\Delta_{\mathbb{Z}^d} : \mathcal{H} \to \mathcal{H}$ be the (negative definite) discrete Laplacian on $\mathbb{Z}^d$, i.e.

$$(\Delta_{\mathbb{Z}^d} u)(n) := \sum_{|n-m|_\infty = 1} (u(m) - u(n)).$$

We define the operator $H_0 : \mathcal{H} \to \mathcal{H}$ by

$$H_0 := -\Delta_{\mathbb{Z}^d} + W,$$

where W is the multiplication operator by a real-valued function, which we also denote by W and which we assume periodic with respect to the subgroup $\gamma := N\mathbb{Z}^d$.

Let $\Box := [0, N-1]^d \subset \mathbb{Z}^d$ and $V^\Box \in \ell^\infty(\mathbb{Z}^d)$ be a non-trivial, compactly supported single-site potential satisfying

$$\mathrm{supp}(V^\Box) \subset \Box.$$