Journal of Functional Analysis 269 (2015) 705–744

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

A generalisation of the form method for accretive forms and operators

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ARTICLE INFO

Article history:
Received 15 October 2014
Accepted 7 April 2015
Available online 8 May 2015
Communicated by L. Gross

MSC:
primary 47A07
secondary 47B44, 46C07, 35J70

Keywords:
Accretive forms
Accretive operators
Form methods
Operator ranges

ABSTRACT

The form method as popularised by Lions and Kato is a successful device to associate m-sectorial operators with suitable elliptic or sectorial forms. McIntosh generalised the form method to an accretive setting, thereby allowing to associate m-accretive operators with suitable accretive forms. Classically, the form domain is required to be densely embedded into the Hilbert space. Recently, this requirement was relaxed by Arendt and ter Elst in the setting of elliptic and sectorial forms.

Here we study the prospects of a generalised form method for accretive forms to generate accretive operators. In particular, we work with the same relaxed condition on the form domain as used by Arendt and ter Elst. We give a multitude of examples for many degenerate phenomena that can occur in the most general setting. We characterise when the associated operator is m-accretive and investigate the class of operators that can be generated. For the case that the associated operator is m-accretive, we study form approximation and Ouhabaz type invariance criteria.

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http://dx.doi.org/10.1016/j.jfa.2015.04.010
0022-1236/

1. Introduction

Lions [14, Theorem 3.6] and Kato [12, Subsection VI.2.1] introduced two different but basically equivalent formulations to generate m-sectorial operators in a Hilbert space H via certain sesquilinear forms. Kato's formulation provides a one-to-one correspondence between m-sectorial operators and closed sectorial forms. In Lions' formulation, closed sectorial forms are replaced by continuous sesquilinear forms whose form domain is a Hilbert space embedded in H and that satisfy an ellipticity condition. This has been generalised in two directions. McIntosh [16] studied accretive forms where the form domain is a Hilbert space embedded in H, and his main aim was to associate an m-accretive operator with such a form. In the other (recent) generalisation by Arendt and ter Elst [1], the form domain is no longer required to be embedded in the Hilbert space H, but a continuous (not necessarily injective) linear map from the form domain into H suffices, together with an ellipticity condition.

The aim of this paper is to give a common generalisation for both [16] and [1], and to study new phenomena that occur in this setting.

We first will give an overview of the generation results mentioned above. In Section 3 we collect basic results about accretive operators. In Section 4 we present our new generation theorem. In Section 5 we give a necessary and sufficient condition in terms of operator ranges for an accretive operator to be associated with a form in the sense of our generation theorem. In Section 6 we give a basic form approximation result. In Section 7 we investigate the relationship between the original form and its dual form. In Section 8 we study a suitable sufficient condition for the range condition in the generation theorem that is adapted from the paper of McIntosh. In Section 9 we investigate the invariance of closed, convex sets under the associated semigroup. Finally, in Section 10 we briefly discuss how our results can be applied in a setting where the form domain is merely a pre-Hilbert space.

Throughout this paper we provide various examples, give implications between many different conditions and highlight fundamental differences to the well-known elliptic theory.

2. Background of the form method

Let V, H be Hilbert spaces, and let $\mathfrak{a}\colon V \times V \to \mathbb{C}$ be a continuous sesquilinear form. Recall that $\mathfrak{a}$ is continuous if and only if there exists an $M \geq 0$ such that $|\mathfrak{a}(u, v)| \leq M\|u\|_V\|v\|_V$ for all $u, v \in V$. If V is continuously and densely embedded in H, then one defines the graph of the **operator A associated with the form $\mathfrak{a}$** in H as follows. Let $x, f \in H$. Then $x \in D(A)$ and $Ax = f$ if and only if $\mathfrak{a}(x, v) = (f \mid v)_H$ for all $v \in V$. Lions [14, Theorem 3.6] proved the following theorem.

Theorem 2.1 *(Lions). Suppose V is continuously and densely embedded in H. Moreover, suppose that $\mathfrak{a}$ is elliptic, i.e., there are $\omega \in \mathbb{R}$ and $\mu > 0$ such that*