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Localization and the Toeplitz algebra on the Bergman space

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ABSTRACT

Let T_f denote the Toeplitz operator with symbol function f on the Bergman space $L^2_a(\mathbf{B}, dv)$ of the unit ball in $\mathbf{C}^n$. It is a natural problem in the theory of Toeplitz operators to determine the norm closure of the set $\{T_f : f \in L^\infty(\mathbf{B}, dv)\}$ in $\mathcal{B}(L^2_a(\mathbf{B}, dv))$. We show that the norm closure of $\{T_f : f \in L^\infty(\mathbf{B}, dv)\}$ actually coincides with the Toeplitz algebra $\mathcal{T}$, i.e., the C^*-algebra generated by $\{T_f : f \in L^\infty(\mathbf{B}, dv)\}$. A key ingredient in the proof is the class of weakly localized operators recently introduced by Isralowitz, Mitkovski and Wick. Our approach simultaneously gives us the somewhat surprising result that $\mathcal{T}$ also coincides with the C^*-algebra generated by the class of weakly localized operators.
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1. Introduction

We begin with a discussion of localized operators. Let $\mathbf{B}$ denote the open unit ball $\{z \in \mathbf{C}^n : |z| < 1\}$ in $\mathbf{C}^n$. The Bergman metric on $\mathbf{B}$ is given by the formula

$$\beta(z, w) = \frac{1}{2} \log \frac{1 + |\varphi_z(w)|}{1 - |\varphi_z(w)|}, \quad z, w \in \mathbf{B},$$

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where φ_z is the Möbius transform of the ball given on [10, page 25]. For each $z \in \mathbf{B}$ and each $r > 0$, the corresponding β-ball will be denoted by $D(z, r)$. That is,

$$D(z, r) = \{w \in \mathbf{B} : \beta(z, w) < r\}.$$

Let dv be the volume measure on $\mathbf{B}$ with the normalization $v(\mathbf{B}) = 1$. Then the formula

$$d\lambda(z) = \frac{dv(z)}{(1 - |z|^2)^{n+1}}$$

gives us the standard Möbius-invariant measure on $\mathbf{B}$.

Recall that the Bergman space $L^2_a(\mathbf{B}, dv)$ is the subspace

$$\{h \in L^2(\mathbf{B}, dv) : h \text{ is analytic on } \mathbf{B}\}$$

of $L^2(\mathbf{B}, dv)$. It is well known that the normalized reproducing kernel for the Bergman space is given by the formula

$$k_z(\zeta) = \frac{(1 - |z|^2)^{(n+1)/2}}{(1 - \langle \zeta, z\rangle)^{n+1}}, \quad z, \zeta \in \mathbf{B}. \tag{1.1}$$

It was first discovered in [14] that *localization* is a powerful tool for analyzing operators on reproducing-kernel Hilbert spaces (more on this in Section 4). Recently, this idea was further explored in [6]. More specifically, in [6] Isralowitz, Mitkovski and Wick introduced the notion of *weakly localized operators* on the Bergman space. Below we give a slightly more refined version of their definition. Our refinement lies in the realization that we can define a class of localized operators for each given localization parameter s.

Definition 1.1. Let a positive number $(n - 1)/(n + 1) < s < 1$ be given.

(a) A bounded operator B on the Bergman space $L^2_a(\mathbf{B}, dv)$ is said to be s-weakly localized if it satisfies the conditions

$$\sup_{z\in\mathbf{B}} \int |\langle Bk_z, k_w\rangle| \left(\frac{1 - |w|^2}{1 - |z|^2}\right)^{s(n+1)/2} d\lambda(w) < \infty,$$

$$\sup_{z\in\mathbf{B}} \int |\langle B^*k_z, k_w\rangle| \left(\frac{1 - |w|^2}{1 - |z|^2}\right)^{s(n+1)/2} d\lambda(w) < \infty,$$

$$\lim_{r\to\infty} \sup_{z\in\mathbf{B}} \int_{\mathbf{B}\setminus D(z,r)} |\langle Bk_z, k_w\rangle| \left(\frac{1 - |w|^2}{1 - |z|^2}\right)^{s(n+1)/2} d\lambda(w) = 0 \quad \text{and}$$

$$\lim_{r\to\infty} \sup_{z\in\mathbf{B}} \int_{\mathbf{B}\setminus D(z,r)} |\langle B^*k_z, k_w\rangle| \left(\frac{1 - |w|^2}{1 - |z|^2}\right)^{s(n+1)/2} d\lambda(w) = 0.$$

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