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Large scale properties for bounded automata groups ☆

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ABSTRACT

In this paper, we study some large scale properties of the mother groups of bounded automata groups. First we give two methods to prove every mother group has infinite asymptotic dimension. Then we study the decomposition complexity of certain subgroup in the mother group. We prove the subgroup belongs to $\mathcal{D}_\omega$.

1. Introduction

Self similar groups (groups generated by automata) were introduced by V.M. Glušhkov [4] in the 1960s, and are now very important in different aspects of mathematics. They are generated by simple automata, but their structures are very complicated and they possess a lot of interesting properties which are hard to find in classical ways. These properties help to answer some famous problems in the early times. For example, the Grigorchuk group [7] can be defined by an automaton with five states over two letters.

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It is the first example of a group with intermediate growth [6], which answered the Milnor problem, and it is also a finitely generated infinite torsion group [5], which answered one of the Burnside problems.

The class of bounded automata groups is a special kind of self similar groups with relatively simple structures, which has been first defined and studied by S. Sidki [12, 13]. This class is very large and it contains most of the well-studied groups, like the Grigorchuk group, the Gupta–Sidki group [9], the Basilica group and so on. S. Sidki proved the structure theorem of bounded automata groups in [12], which describes how elements in them look like. Recently an embedding theorem has been proven [1] which said that there exists a series of mother groups such that every finitely generated bounded automata group can be embedded into one of them. And it has also been proven that mother groups are amenable, so is any bounded group.

In this paper, we study two large scale properties of the mother groups: asymptotic dimension and finite decomposition complexity. Asymptotic dimension was firstly introduced by Gromov in 1993 as a coarse analogue of the classical topological covering dimension, but it didn't get much attention until G. Yu in 1998 proved that the Novikov higher signature conjecture holds for groups with finite asymptotic dimensions [16]. So it is important to study whether the mother groups have finite asymptotic dimensions or not. In [14] J. Smith has proved that the Grigorchuk group has infinite asymptotic dimension, then by the embedding theorem, most of the mother groups have infinite asymptotic dimensions, except several ones with fewer letters. We prove:

Main Theorem 1. *All of the mother groups G_d of bounded automata groups have infinite asymptotic dimensions for $d > 2$.*

We prove this theorem by two different methods. One is to show the mother group G_3 is coarsely equivalent to the cubic power of itself. Another is more precise: we show that the direct sum of countable infinitely many copies of integer can be embedded into all of the mother groups G_d for $d > 2$.

Next, we study the decomposition complexity of the mother group G_3. Finite decomposition complexity (FDC) is a concept introduced by E. Guentner, R. Tessera and G. Yu [8] in order to solve certain strong rigidity problem including the stable Borel conjecture. It generalizes finite asymptotic dimension. Briefly speaking, a metric space has FDC if it admits an algorithm to decompose itself into some nice pieces which are easy to handle in certain asymptotic way. We focus on the decomposition complexity of a special subgroup in the mother group G_3. It was derived naturally from the proof of the first main theorem. We study the commutative relations between the generators, then use induction to prove this subgroup belongs to $\mathcal{D}_\omega$ where ω is the first infinite ordinal number. In particular, this subgroup has FDC. The notion $\mathcal{D}_\omega$ will be introduced in the next section.