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Bessel functions, heat kernel and the conical Kähler–Ricci flow

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ABSTRACT

Following Donaldson's openness theorem on deforming a conical Kähler–Einstein metric, we prove a parabolic Schauder-type estimate with respect to conical metrics. As a corollary, we show that the conical Kähler–Ricci flow exists for short time. The key is to establish the relevant heat kernel estimates, where we use the Weber formula on Bessel function of the second kind and Carslaw's heat kernel representation in [8].

Contents

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1. Introduction

Fix $\beta \in (0,1)$. The simplest conic metric is written as

$$\eta_\beta = dr^2 + \beta^2 r^2 d\theta^2, \quad \theta \in [0, 2\pi),\ r \in [0, \infty). \tag{1}$$

This gives a metric on the complex plane $\mathbb{C}$, with a cone point of angle $2\beta\pi$ at the origin. This cone is obtained by attaching the two sides of a wedge of angle $2\beta\pi$ (see Fig. 1). In particular, η_1 is exactly the Euclidean metric.

Using the holomorphic coordinate $z_0 = r^{\frac{1}{\beta}} e^{\sqrt{-1}\theta}$, we get

$$\eta_\beta = \frac{\beta^2}{|z_0|^{2-2\beta}} dz_0 \otimes d\bar{z}_0.$$

The Kähler (sympletic) form of η_β is $\frac{\beta^2}{|z_0|^{2-2\beta}} \sqrt{-1} dz_0 \wedge d\bar{z}_0$.

Based on the above low dimensional pictures, we can consider conic metrics in general dimensions which are singular along analytic hypersurfaces. The model cone metric over $\mathbb{C} \times \mathbb{C}^{n-1}$ which is singular along the analytic hypersurface $\{0\} \times \mathbb{C}^{n-1}$ is

$$\omega_\beta = \frac{\beta^2}{|z_0|^{2-2\beta}} \sqrt{-1} dz_0 \wedge d\bar{z}_0 + \sum_{j=1}^{n-1} \sqrt{-1} dz_j \wedge d\bar{z}_j, \quad n \geq 1. \tag{2}$$

Let $m = 2n - 2$, then the metric tensor of ω_β in the polar coordinates is

$$g_\beta = dr^2 + \beta^2 r^2 d\theta^2 + g_{\mathbb{R}^m}, \tag{3}$$

where $g_{\mathbb{R}^m} = \sum_{i=1}^m ds_i^2$. Henceforth we will mainly use the notation $\mathbb{R}^m$ instead of $\mathbb{C}^n$, as our Schauder estimates are with respect to the real setting.

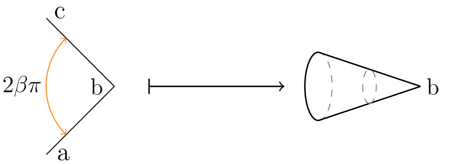

Fig. 1. The simplest cone.