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Sobolev spaces with variable exponents on complete manifolds

Michał Gaczkowski [a], Przemysław Górka [a,*], Daniel J. Pons [b,*]

[a] *Department of Mathematics and Information Sciences, Warsaw University of Technology, Ul. Koszykowa 75, 00-662 Warsaw, Poland*
[b] *Facultad de Ciencias Exactas, Departamento de Matemáticas, Universidad Andres Bello, República 220, Santiago, Chile*

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ABSTRACT

We study variable exponent function spaces on complete non-compact Riemannian manifolds. Using classic assumptions on the geometry, continuous embeddings between Sobolev and Hölder function spaces are obtained. We prove compact embeddings of H-invariant Sobolev spaces, where H is a compact Lie subgroup of the group of isometries of the manifold. As an application, we prove the existence of non-trivial solutions to non-homogeneous $q(x)$-Laplace equations.
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1. Introduction

In Euclidean domains, the theory of Lebesgue–Sobolev and Hölder spaces with variable exponents has applications in non-linear elastic mechanics [28], electrorheological fluids

* Corresponding authors.
E-mail addresses: p.gorka@mini.pw.edu.pl (P. Górka), dpons@unab.cl, pons.dan@gmail.com (D.J. Pons).

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[24], and image restoration [20], for example. In those areas of science, the exponential growth of quantities induced by observables is modified by spatial inhomogeneities of the material, fluid and image, respectively, or by external perturbations, usually modeled as multiplicative and additive factors. In improved models for such phenomena, the exponents describing the growth are allowed to vary in space according to some law, see [20,24].

As a consequence, Lebesgue–Sobolev spaces with variable exponents have been studied in depth during the last decade (see the surveys [8,25]). In Euclidean space, Sobolev type inequalities with variable exponents have been delivered [6,10,15]. This theory has been extended to metric measure spaces [12,16], and to Riemannian manifolds, see [13].

In geometric and global analysis, classic Sobolev and Hölder spaces on Riemannian manifolds have been studied for more than fifty years [23,2,17]. They have been used to obtain isoperimetric type inequalities [17], and in the Yamabe problem for conformal metrics with prescribed scalar curvature [27], among other topics (see [2] for a survey).

In this article we deal with Sobolev and Hölder spaces with variable exponents on complete non-compact Riemannian manifolds. The applications mentioned above suggest that this setup should be useful for similar problems with an additional geometric flavor.

We organize this work in several sections. In Section 2 we give definitions and results that will be used later on. Section 3 is a brief introduction to variable Sobolev and Hölder spaces on Riemannian manifolds. Our main contributions begin in Section 4. In Section 4.1, assuming continuity of the exponents and bounds on the geometry of the manifold, we obtain embeddings between Sobolev spaces. Then in Section 4.2, using stronger bounds on the geometry of the space, and also stronger hypothesis on the exponents, namely log-Hölder continuity, we obtain the embedding between Sobolev spaces with critical exponents, and embed Sobolev spaces in Hölder spaces. In Section 5 we consider H-invariant Sobolev spaces, where H is a subgroup of the group of isometries of the manifold (M, g); this leads to compact embeddings. A brief discussion of the conditions that H and g should satisfy provides a huge family of examples. Finally, in Section 6, we show the existence of non-trivial H-invariant solutions to a non-linear elliptic equation involving the $q(x)$-Laplacian.

2. Preliminaries

2.1. Variable exponent Lebesgue spaces

We recall some facts and notation about variable exponent Lebesgue and Sobolev spaces. Most of the properties of these spaces can be found in [7] and [18].

Let (Ω, μ) be a σ-finite, complete measure space. By a variable exponent we mean a bounded measurable function $q : \Omega \to [1, \infty]$. We denote by $\mathcal{P}(\Omega)$ the set of variable exponents on Ω. Given q in $\mathcal{P}(\Omega)$ and $A \subset \Omega$, define

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