Journal of Functional Analysis 270 (2016) 1602–1621

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Multiple sign changing solutions for semi-linear corner degenerate elliptic equations with singular potential ☆

Hua Chen [a], Shuying Tian [a,*], Yawei Wei [b]

[a] *School of Mathematics and Statistics, Wuhan University, Wuhan 430072, China*
[b] *School of Mathematical Sciences and LPMC, Nankai University, Tianjin 300071, China*

ARTICLE INFO

Article history:
Received 30 June 2015
Accepted 12 November 2015
Available online 3 December 2015
Communicated by C. De Lellis

Keywords:
Corner-degenerate elliptic operators
Singular potential
Corner type Hardy inequality
Sign changing solution

ABSTRACT

We consider the Dirichlet problem of semi-linear corner-degenerate elliptic equations with singular potentials in the subcritical case. First we introduce the weighted corner type Sobolev inequality, Poincaré inequality and Hardy inequality. Then, by using the variational method and the abstract theory for sign changing solution, we obtain the existence of infinitely many sign changing solutions in the weighted corner Sobolev space.

1. Introduction

Let $\mathbb{M} \subset [0,1) \times X \times [0,1)$ be a corner type domain with finite corner measure $|\mathbb{M}| = \int_{\mathbb{M}} \frac{dr}{r} dx \frac{dt}{rt}$, which is a local model of stretched corner-manifolds (*i.e.* the manifolds

☆ This work is Supported by National Natural Science Foundation of China (Grant Nos. 11131005, 11171261).

* Corresponding author.
E-mail addresses: chenhua@whu.edu.cn (H. Chen), sytian@whu.edu.cn (S. Tian), weiyawei@nankai.edu.cn (Y. Wei).

http://dx.doi.org/10.1016/j.jfa.2015.11.007
0022-1236/

with corner singularities) with dimension $N = n + 2 \geq 3$. Here X is a closed compact sub-manifold of dimension n embedded in the unit sphere of $\mathbb{R}^{n+1}$. Let $\mathbb{M}_0$ denote the interior of $\mathbb{M}$ and $\partial\mathbb{M} = \{0\} \times X \times \{0\}$ denote the boundary of $\mathbb{M}$. The corner-Laplacian is defined as

$$\triangle_{\mathbb{M}} = (r\partial_r)^2 + (\partial_{x_1})^2 + \ldots + (\partial_{x_n})^2 + (rt\partial_t)^2,$$

which is a degenerate elliptic operator on the boundary $\partial\mathbb{M}$. Such kinds of degenerate operators have been studied by a lot of authors, e.g. [7,10–12]. In [3–5,7], Chen–Liu–Wei introduced the corner type weighted p-Sobolev spaces, discussed the properties of continuous embedding, compactness and spectrum, and then proved the corner type Sobolev inequality and Poincaré inequality. They also considered the existence of multiple weak solutions for semi-linear equations with the corner-Laplacian. There is no result about the corner Hardy inequality which is crucial in studying the equations with potential. In the present paper, the authors discuss more information about the solution, for example, whether the solution changes sign or not. Here we study the following Dirichlet problem with singular potentials where the nonlinear term is $\lambda u + |u|^{p-2}u$.

$$\begin{cases} -\triangle_{\mathbb{M}} u - \mu V u = \lambda u + |u|^{p-2}u & z := (r, x, t) \in \mathbb{M}_0, \\ u = 0 & \text{on } \partial\mathbb{M}. \end{cases} \tag{1.1}$$

Here $\lambda > 0$, $0 < \mu < 1$, $2 < p < 2^*$ and the singular potential function V, unbounded over $\partial\mathbb{M}$, satisfies

$$\|V^{\frac{1}{2}} u\|_{L_2^{\frac{N-1}{2},\frac{N}{2}}(\mathbb{M})} \leq \|\nabla_{\mathbb{M}} u\|_{L_2^{\frac{N-1}{2},\frac{N}{2}}(\mathbb{M})}. \tag{1.2}$$

Our main results can be stated as follows.

Theorem 1.1. *Under the conditions above, for any $\lambda > 0$, the Dirichlet problem (1.1) has infinitely many sign changing solutions in the weighted Sobolev space $\mathcal{H}_{2,0}^{1,(\frac{N-1}{2},\frac{N}{2})}(\mathbb{M})$.*

With the help of Theorem 1.1, we can also consider the sign changing solution of the following problem by a perturbation method.

$$\begin{cases} -\triangle_{\mathbb{M}} u - \mu V u = \lambda u + |u|^{p-2}u + \beta f(z, u) & z := (r, x, t) \in \mathbb{M}_0, \\ u = 0 & \text{on } \partial\mathbb{M}, \end{cases} \tag{1.3}$$

where we assume

(A). $f \in C(\mathbb{M} \times \mathbb{R}, \mathbb{R})$, $f(z, s)s \geq 0$ for all $z \in \mathbb{M}$ and $s \in \mathbb{R}$; $\lim\limits_{s\to 0} \dfrac{f(z, s)}{s} = 0$ uniformly in $z \in \mathbb{M}$.

Then we have

Corollary 1.1. *If the function f satisfies the condition (A) above, then the Dirichlet problem (1.3) has infinitely many sign changing solutions in $\mathcal{H}_{2,0}^{1,(\frac{N-1}{2},\frac{N}{2})}(\mathbb{M})$.*