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Mittag-Leffler analysis I: Construction and characterization

M. Grothaus [a], F. Jahnert [a,*], F. Riemann [a], J.L. da Silva [b]

[a] *Department of Mathematics, University of Kaiserslautern, P.O. Box 3049, 67653 Kaiserslautern, Germany*
[b] *Universidade da Madeira, Edifício da Penteada, Caminho da Penteada, 9020-105, Funchal, Madeira, Portugal*

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ABSTRACT

We construct an infinite dimensional analysis with respect to non-Gaussian measures of Mittag-Leffler type which we call Mittag-Leffler measures. It turns out that the well-known Wick ordered polynomials in Gaussian analysis cannot be generalized to this non-Gaussian case. Instead of using Wick ordered polynomials we prove that a system of biorthogonal polynomials, called Appell system, is applicable to the Mittag-Leffler measures. Therefore we are able to introduce a test function and a distribution space. As an application we construct Donsker's delta in a non-Gaussian setting as a weak integral in the distribution space.

* Corresponding author.
E-mail addresses: grothaus@mathematik.uni-kl.de (M. Grothaus), jahnert@mathematik.uni-kl.de (F. Jahnert), riemann@mathematik.uni-kl.de (F. Riemann), luis@uma.pt (J.L. da Silva).

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1. Introduction

During the last decades white noise analysis has evolved into an infinite dimensional distribution theory, with rapid developments in mathematical structure and applications in various domains. For an overview we refer to the monographs [18,39,28]. Especially, a deep understanding of the structure of spaces of smooth and generalized random variables over the white noise space or, more generally, Gaussian spaces is provided by various characterization theorems [42,24,16]. Also, the theory of white noise analysis and the including tools can be applied to a number of fields in mathematics and physics, for example Feynman integration [9,51,22,29], representation of quantum field theory [5,43], intersection local times for Brownian motion [8,52] as well as for fractional Brownian motion [11,40], Dirichlet forms [3,4,19], infinite dimensional harmonic analysis [17] and so forth.

Almost at the same time, first attempts were made to introduce a non-Gaussian infinite dimensional analysis, by transferring properties of the Gaussian measure to the Poisson measure [20]. This approach can be generalized with the help of a biorthogonal system, which consists of generalized Appell systems [7,2,27]. It was shown that this approach is suitable for a wide class of measures, including the Gaussian measure and the Poisson measure [25]. Two properties of the measure are essential: An analyticity condition of its Laplace transform and a non-degeneracy or positivity condition (see also [21]). In this concept, similar notions and characterizations as in Gaussian analysis can be introduced [27].

In this work we concentrate on those measures whose characteristic functions are given via Mittag-Leffler functions. We refer to these measures as Mittag-Leffler measures. The grey noise measure [49,38] is included as a special case in the class of Mittag-Leffler measures, which offers the possibility to apply the Mittag-Leffler analysis to fractional differential equations, in particular to fractional diffusion equations [48,49,32], which carry numerous applications in science, like relaxation type differential equations or viscoelasticity.

In this paper we prove that the Mittag-Leffler measures belong to the class of measures for which Appell systems exist. Hence we are able to introduce spaces of test functions and distributions with respect to Mittag-Leffler measures. Moreover we construct a distribution which can be considered as a generalization of Donsker's delta from Gaussian analysis.

2. The finite dimensional Mittag-Leffler measure

In this section we recall the Mittag-Leffler measures in the finite dimensional Euclidean space $\mathbb{R}^d$, $d \in \mathbb{N}$. Using the Gram–Schmidt method we compute the first orthogonal polynomials for that measure and show certain key properties of them.

Definition 2.1. For $0 < \beta < \infty$ the Mittag-Leffler function is an entire function defined by its power series