Journal of Functional Analysis 268 (2015) 2851–2905

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Asymptotics of unitary multimatrix models: The Schwinger–Dyson lattice and topological recursion

Alice Guionnet [1], Jonathan Novak *

Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02139, United States

A R T I C L E I N F O

Article history:
Received 12 February 2014
Accepted 2 March 2015
Available online 25 March 2015
Communicated by A. Borodin

Keywords:
Matrix models
Random matrices
Concentration of measure
Noncommutative probability

A B S T R A C T

We prove the existence of a $1/N^2$ expansion in unitary multimatrix models which are Gibbs perturbations of the Haar measure, and express the expansion coefficients recursively in terms of the unique solution of a noncommutative initial value problem. The recursion obtained is closely related to the "topological recursion" which underlies the asymptotics of many random matrix ensembles and appears in diverse enumerative geometry problems. Our approach consists of two main ingredients: an asymptotic study of the Schwinger–Dyson lattice over noncommutative Laurent polynomials, and uniform control on the cumulants of Gibbs measures on product unitary groups. The required cumulant bounds are obtained by concentration of measure arguments and change of variables techniques.

Contents

* Corresponding author.
E-mail addresses: guionnet@math.mit.edu (A. Guionnet), jnovak@math.mit.edu (J. Novak).

[1] Research partially supported by the Simons Foundation and NSF award DMS-1307704.

http://dx.doi.org/10.1016/j.jfa.2015.03.002

1. Introduction

1.1. *A noncommutative initial value problem*

1.1.1. Given a unital $*$-algebra B defined over $\mathbb{C}$, let

$$L = B\langle u_1^{\pm 1}, \dots, u_m^{\pm 1}\rangle$$

denote the algebra of Laurent polynomials in m noncommutative variables $u_1, \dots, u_m$, with noncommutative coefficients in B. That is,

$$L = B * \mathbb{C}\langle u_1^{\pm 1}, \dots, u_m^{\pm 1}\rangle,$$

the free product of B and the group algebra of a free group of rank m.