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Classification of isolated singularities for nonhomogeneous operators in divergence form

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A B S T R A C T

Consider the equation $\operatorname{div}(\frac{\varphi(|\nabla u|)}{|\nabla u|}\nabla u) = 0$ on the punctured unit ball from $\mathbb{R}^N$ $(N \geq 2)$, where φ is an odd, increasing homeomorphism from $\mathbb{R}$ onto $\mathbb{R}$ of class C^1. Under reasonable assumptions on φ we prove that if u is a non-negative solution of our equation, then either 0 is a removable singularity of u or u behaves near 0 as the fundamental solution of the equation investigated here. In particular, our result complements to the case on nonhomogeneous operators in divergence form Bôcher's Theorem and some classical results by Serrin.

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1. Introduction

The goal of this paper is to give a complete classification of isolated singularities in the punctured unit ball, i.e. $B^\star := B_1(0) \setminus \{0\} \subset \mathbb{R}^N$ ($N \geq 2$), for the equation

$$\operatorname{div}\left(\frac{\varphi(|\nabla u|)}{|\nabla u|}\nabla u\right) = 0, \quad x \in B^\star, \tag{1.1}$$

where φ is an odd, increasing homeomorphism from $\mathbb{R}$ onto $\mathbb{R}$ of class C^1 satisfying

$$0 < \delta \leq \frac{t\varphi'(t)}{\varphi(t)} \leq \varphi_0 < N - 1, \quad \forall t \geq 0, \tag{1.2}$$

for certain constants δ and φ_0 for which $0 < \delta \leq \varphi_0 < N - 1$.

The problem of studying the behavior of solutions for some prescribed PDE's in the neighborhood of a singularity has a long history. Without having the pretension to outline all the fundamental results on this topic let us recall a few known facts of great impact in the field and which are directly related to equations of type (1.1). With that end in view, we start by turning back to a fundamental result from harmonic analysis, namely Bôcher's Theorem (see [2]): *any positive harmonic function in the punctured unit ball from $\mathbb{R}^N$ ($N \geq 2$), i.e. $B_1(0) \setminus \{0\}$, can be represented as a linear combination of a harmonic function in the whole unit ball with a fundamental solution of the Laplace operator*. Unfortunately, such complete results as Bôcher's Theorem cannot be formulated in the case when in the considered equation a nonlinearity is involved. However, some results of great impact concerning the classification of isolated singularities of solutions for quasilinear elliptic equations in divergence form were given by Serrin in [15,16]. To be more precise, the equations investigated by Serrin have the form

$$\operatorname{div} \mathbb{A}(x, u, \nabla u) = B(x, u, \nabla u), \tag{1.3}$$

where $\mathbb{A} = \mathbb{A}(x, u, \xi)$ is a given vector function and $B = B(x, u, \xi)$ is a given scalar function such that the growth of $\mathbb{A}$ dominates the growth of B. Letting $B = 0$ in (1.3) the basic assumptions on $\mathbb{A} = \mathbb{A}(x, u, \xi)$ in [15,16] are the following

$$\begin{gathered} \left|\mathbb{A}(x, u, \xi)\right| \leq a|\xi|^{p-1} + b|u|^{p-1} + e \\ \left\langle \xi, \mathbb{A}(x, u, \xi)\right\rangle \geq |\xi|^{p-1} - d|u|^{p-1} - g, \end{gathered} \tag{1.4}$$

where $1 < p \leq N$, $a \in \mathbb{R}^+$, b, d, e and g are measurable functions of x such that

$$b, e \in L^{\frac{N}{p-1-\epsilon}}, \qquad d, g \in L^{\frac{N}{p-\epsilon}}.$$

Let $\mu(x)$ denote a fundamental solution of Eq. (1.3), i.e. a solution of

$$\operatorname{div} \mathbb{A}(x, \mu, \nabla\mu) = \delta_0 \quad \text{in } \mathcal{D}'\left(\mathbb{R}^N\right),$$