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Interpolation of compact operators by general interpolation methods

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ABSTRACT

Let a linear operator T act compactly from a Banach couple $\vec{X} := (X_0, X_1)$ in a Banach lattice couple $\vec{B} := (B_0, B_1)$. If both B_i have absolutely continuous norm, then any interpolation functor F preserves two-sided compactness of T. The same holds if only one of B_i satisfies the above condition and F is a quasipower regular functor. The similar results are true for operators acting from $\vec{B}$ in $\vec{X}$ under the above formulated assumptions for simultaneously $\vec{B}$ and its dual $\vec{B}'$.

1. Introduction

The following problem is going back to the classical papers of Lions–Peetre [14] and Calderón [3] of the sixties. In a generalized form, it can be formulated as follows.

Let F be an *interpolation functor*[1] and T be a linear operator acting continuously between *Banach couples* $\vec{X} := (X_0, X_1)$ and $\vec{Y} := (Y_0, Y_1)$. Assume that it acts compactly from X_i in Y_i, for $i \in \{0, 1\}$ (*two-sided compactness*) or for a fixed i (*one-sided compactness*).

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[1] The concepts singled out by italic will be discussed in Section 2; see also the books [1] and [2].

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Problem. Under what conditions on the $F, \vec{X}, \vec{Y}$ the operator T acts compactly between interpolation spaces $F(\vec{X})$ and $F(\vec{Y})$?

In this paper, both versions of the question will be answered positively when one of the Banach couples consists of a pair of *Banach lattices*, say $\vec{B} := (B_0, B_1)$, on a σ-finite measure space.

In the case of two-sided compactness, we assume that each B_i has *absolutely continuous norm* if $\vec{Y} = \vec{B}$ and assume that also the *dual* Banach lattices B_i', $i \in \{0, 1\}$, have this property if $\vec{X} = \vec{B}$. Moreover, interpolation functor F is arbitrary for the first case and *regular* for the second.

In the case of one-sided compactness, we impose the above formulated conditions only on one of B_i; however, both B_i are assumed to be *symmetric*. The conditions on F are the same but, in addition, F is assumed to be *quasipower*.

The problem has been actively studying since the 60s up to the present time. It was answered positively for a large variety of special cases including *the Lions–Peetre interpolation functor* $(\cdot)_{\theta p}$, $0 < \theta < 1$, see [8] and [7], and a more general real methods, see [6] and the 16 previous papers referring to therein.

Unlike these papers dealing with arbitrary Banach couples, the problem for the Calderón complex methods was solved only for a special choices of $\vec{X}$ or/and $\vec{Y}$, see [10], the 12 papers, and the recent website referred to therein and the paper [9]. We point out as a typical result of [9] where $\vec{Y} = \vec{B}$ with B_i's being of absolutely continuous norm or having the *Fatou property*.

Up to our knowledge there are only the next three papers dealing with general interpolation functors. The papers [17] and [5] study the case of Banach couples $\vec{X}, \vec{Y}$ one of which is "diagonal", i.e., $Y_0 = Y_1$. The third paper [18] studies the case of a general quasipower function F acting from arbitrary $\vec{X}$ into $\vec{Y} = \vec{B}$ with B_i's having absolutely continuous norm.

Since the results of the first two papers are formulated without using the interpolation content and the proof in the third is incomplete, we will discuss them in the concluding Section 6.

The paper is organized as follows.

In Section 2, one recalls the required definitions and prove few auxiliary results; it also consists of all notations using throughout the following text. The main four theorems are formulated in Section 3 while their proofs are presented in Sections 4 and 5 devoted, respectively, to two- and one-sided compactness cases.

The concluding Section 6 discusses the aforementioned three papers.

Remark. Throughout the paper we deal only with real Banach spaces and lattices.

2. Preliminaries

Banach lattices. Let (Ω, Σ, μ) be a σ-finite measure space (here and in the sequel briefly denoted by Ω).