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A diffusive logistic equation with a free boundary and sign-changing coefficient in time-periodic environment ☆

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ABSTRACT

This paper concerns a diffusive logistic equation with a free boundary and sign-changing intrinsic growth rate in heterogeneous time-periodic environment, in which the variable intrinsic growth rate may be "very negative" in a "suitable large region" (see conditions **(H1)**, **(H2)**, (4.3)). Such a model can be used to describe the spreading of a new or invasive species, with the free boundary representing the expanding front. In the case of higher space dimensions with radial symmetry and when the intrinsic growth rate has a positive lower bound, this problem has been studied by Du, Guo & Peng [11]. They established a spreading–vanishing dichotomy, the sharp criteria for spreading and vanishing and estimate of the asymptotic spreading speed. In the present paper, we show that the above results are retained for our problem.

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1. Introduction

In the real world, the invasion of harmful species and/or introduction of new and beneficial species are natural phenomena. It is of primary importance to predict and analyze the growing and spreading mechanism of biological invasions. A lot of mathematicians have made efforts to develop various invasion models and investigated them from a viewpoint of mathematical ecology. Most theoretical approaches are based on or start with single-species models. In consideration of the heterogeneous environment, the following problem

$$\begin{cases} u_t - d\Delta u = a(t,x)u - b(t,x)u^2, & t > 0, \quad x \in \Omega, \\ B[u] = 0, & t \geq 0, \quad x \in \partial\Omega, \\ u(0,x) = u_0(x), & x \in \Omega \end{cases}$$

is a typical one to describe the spread, persistence and extinction of the new or invasive species and has received an astonishing amount of attention, please refer to [4,5,7] and [17,19,20,25,31] for example. In this model, $u(t,x)$ represents the population density; constant $d > 0$ denotes the diffusion (dispersal) rate; $a(t,x)$ and $b(t,x)$ represent the intrinsic growth rate and self-limitation coefficient of the species, respectively; Ω is a bounded domain of $\mathbb{R}^N$; the boundary operator $B[u] = \alpha u + \beta\frac{\partial u}{\partial \nu}$, α and β are non-negative functions and satisfy $\alpha + \beta > 0$, ν is the outward unit normal vector of the boundary $\partial\Omega$. The corresponding systems with heterogeneous environment have also been studied extensively, please refer to [5,7,21,25,31] and the references cited therein.

In most spreading processes in the natural world, a spreading front can be observed. When a new or invasive species initially occupies a region Ω_0 with density $u_0(x)$, as time t increases, it is natural to expect that Ω_0 will evolve into an expanding region $\Omega(t)$ with an expanding front $\partial\Omega(t)$, inside which the initial function $u_0(x)$ will evolve into a positive function $u(t,x)$ governed by a suitable diffusive equation, with $u(t,x)$ vanishing on the moving boundary $\partial\Omega(t)$.

In the natural world, for most animals and plants, their birth and death rates will change with the seasons, so the intrinsic growth rate $a(t,x)$ and then the self-limitation coefficient $b(t,x)$ should be time-periodic functions. Especially, in the winter of severe cold and cold zones, animals cannot capture enough food to feed upon and do not breed, seeds cannot germinate and buds cannot grow above ground, so their birth rates are zero. In the meantime, their death rates will be greater. Therefore, in some periods and some areas, the intrinsic growth rate $a(t,x)$ may be negative. In order to simplify the mathematics, in this paper we only consider the one dimensional case, i.e., $N = 1$, and assume that the left boundary is fixed: $x = 0$. For a more realistic description of the growth mechanism and spreading of a new or invasive species, throughout this paper we assume