Journal of Functional Analysis 270 (2016) 854–859

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Corrigendum

Corrigendum to "Classifying C^*-algebras with both finite and infinite subquotients" [J. Funct. Anal. 265 (2013) 449–468]

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ARTICLE INFO

Article history:
Received 18 June 2015
Accepted 30 September 2015
Available online 10 November 2015
Communicated by Stefaan Vaes

MSC:
primary 46L35, 37B10
secondary 46M15, 46M18

Keywords:
Classification
Extensions
Graph algebras

ABSTRACT

As recently pointed out by Gabe, a fundamental paper by Elliott and Kucerovsky concerning the absorption theory for C^*-algebras contains an error, and as a consequence we must report that Lemma 4.5 in [3] is not true as stated. In this corrigendum, we prove an adjusted statement and explain why the error has no consequences to the main results of [3]. In particular, it is noted that all the authors' claims concerning Morita equivalence or stable isomorphism of graph C^*-algebras remain correct as stated.

In this note, we give a counterexample to [3, Lemma 4.5] and we make the necessary changes to make the statement true. Before doing this, we first explain where the error

DOI of original article: http://dx.doi.org/10.1016/j.jfa.2013.05.006.
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http://dx.doi.org/10.1016/j.jfa.2015.09.027
0022-1236/

occurred. In the proof of [3, Lemma 4.5] we used [6, Corollary 16] to conclude that a non-unital, purely large extension is nuclearly absorbing. This was the key component to prove [3, Lemma 4.5]. However, it was recently pointed out by James Gabe in [7] that [6, Corollary 16] is false in general; Gabe showed that there exists a non-unital extension that is purely large but not nuclearly absorbing. The error occurs for non-unital extensions $0 \to \mathfrak{I} \to \mathfrak{E} \to \mathfrak{A} \to 0$ with $\mathfrak{A}$ unital. We can use [7, Example 1.1], to find a counterexample to [3, Lemma 4.5] as follows:

Example 1. Let p be a projection in $\mathbb{B}(\ell^2)$ such that p and $1_{\mathbb{B}(\ell^2)} - p$ are norm-full, properly infinite projections in $\mathbb{B}(\ell^2)$. Let $\mathfrak{e}\colon 0 \to \mathbb{K} \oplus \mathbb{K} \to \mathfrak{E} \to \mathbb{C} \to 0$ be the trivial extension induced by the $*$-homomorphism which maps $\lambda \in \mathbb{C}$ to $\lambda(p \oplus 1_{\mathbb{B}(\ell^2)})$. Since p and $1_{\mathbb{B}(\ell^2)} - p$ are norm-full, properly infinite projections in $\mathbb{B}(\ell^2)$, we have that p and $1_{\mathbb{B}(\ell^2)} - p$ are not elements of $\mathbb{K}$. Therefore, $1_{\mathbb{B}(\ell^2)} \oplus 1_{\mathbb{B}(\ell^2)} - p \oplus 1_{\mathbb{B}(\ell^2)} = (1_{\mathbb{B}(\ell^2)} - p) \oplus 0$ is not an element of $\mathbb{K} \oplus \mathbb{K}$. Hence, $\mathfrak{e}$ is a non-unital extension. By [7, Example 1.1], $\mathfrak{e}$ is a purely large, full extension that is not nuclearly absorbing. Therefore, $\mathfrak{e}$ is not absorbing since $\mathbb{C}$ is a nuclear C^*-algebra. Therefore, $\mathfrak{e}$ cannot be isomorphic to an absorbing extension.

We now construct a non-unital, absorbing extension $\mathfrak{f}\colon 0 \to \mathbb{K} \oplus \mathbb{K} \to \mathfrak{F} \to \mathbb{C} \to 0$ such that $[\tau_{\mathfrak{e}}] = [\tau_{\mathfrak{f}}]$ in $\mathrm{KK}^1(\mathbb{C}, \mathbb{K} \oplus \mathbb{K})$, where $\tau_{\mathfrak{e}}$ and $\tau_{\mathfrak{f}}$ are the Busby invariants of $\mathfrak{e}$ and $\mathfrak{f}$ respectively. Let q be a projection in $\mathbb{B}(\ell^2)$ such that q and $1_{\mathbb{B}(\ell^2)} - q$ are norm-full, properly infinite projections in $\mathbb{B}(\ell^2)$. Let $\mathfrak{f}\colon 0 \to \mathbb{K} \oplus \mathbb{K} \to \mathfrak{F} \to \mathbb{C} \to 0$ be the trivial extension induced by the $*$-homomorphism which maps $\lambda \in \mathbb{C}$ to $\lambda(p \oplus q)$. Using a similar argument as in the case for $\mathfrak{e}$, we have that $\mathfrak{f}$ is a non-unital extension. By construction, $\mathfrak{f}$ is a full extension and hence, $\mathfrak{f}$ is a purely large extension since $\mathbb{K} \oplus \mathbb{K}$ has the corona factorization property. Since $1_{\mathbb{B}(\ell^2)} - p$ and $1_{\mathbb{B}(\ell^2)} - q$ are norm-full, properly infinite projections in $\mathbb{B}(\ell^2)$, we have that $1_{\mathbb{B}(\ell^2)} \oplus 1_{\mathbb{B}(\ell^2)} - p \oplus q = (1_{\mathbb{B}(\ell^2)} - p) \oplus (1_{\mathbb{B}(\ell^2)} - q)$ is a norm-full, properly infinite projection in $\mathbb{B}(\ell^2) \oplus \mathbb{B}(\ell^2)$. Moreover, we have that $(1_{\mathbb{B}(\ell^2)} \oplus 1_{\mathbb{B}(\ell^2)} - p \oplus q)\mathfrak{F} \subseteq \mathbb{K} \oplus \mathbb{K}$. Therefore, by [7, Theorem 2.3], $\mathfrak{f}$ is a nuclearly absorbing extension, and hence absorbing since $\mathbb{C}$ is nuclear. Since $\mathfrak{e}$ and $\mathfrak{f}$ are trivial extensions, we have that $[\tau_{\mathfrak{e}}] = [\tau_{\mathfrak{f}}] = 0$ in $\mathrm{KK}^1(\mathbb{C}, \mathbb{K} \oplus \mathbb{K})$. Thus we have proved the existence of $\mathfrak{f}$.

Since $\mathfrak{e}$ is not an absorbing extension and $\mathfrak{f}$ is an absorbing extension, we have that $\mathfrak{e}$ is not isomorphic to $\mathfrak{f}$. Note that

$$\mathrm{KK}(\mathrm{id}_{\mathbb{C}}) \times [\tau_{\mathfrak{f}}] = [\tau_{\mathfrak{f}}] = [\tau_{\mathfrak{e}}] = [\tau_{\mathfrak{e}}] \times \mathrm{KK}(\mathrm{id}_{\mathbb{K}\oplus\mathbb{K}})$$

in $\mathrm{KK}^1(\mathbb{C}, \mathbb{K} \oplus \mathbb{K})$. We claim that $\mathfrak{E}$ is not isomorphic to $\mathfrak{F}$. Suppose there exists a $*$-isomorphism $\varphi\colon \mathfrak{E} \to \mathfrak{F}$. Let $\pi_{\mathfrak{f}}$ be the canonical surjective $*$-homomorphism from $\mathfrak{F}$ to $\mathbb{C}$. Since φ and $\pi_{\mathfrak{f}}$ are surjective, we have that $(\pi_{\mathfrak{f}} \circ \varphi)(\mathbb{K} \oplus \mathbb{K})$ is an ideal of $\mathbb{C}$. So, $(\pi_{\mathfrak{f}} \circ \varphi)(\mathbb{K} \oplus \mathbb{K}) = 0$ or $(\pi_{\mathfrak{f}} \circ \varphi)(\mathbb{K} \oplus \mathbb{K}) = \mathbb{C}$. Since $\mathbb{K} \oplus \mathbb{K}$ has exactly four ideals, $0, \mathbb{K} \oplus 0, 0 \oplus \mathbb{K}$, and $\mathbb{K} \oplus \mathbb{K}$, we have that $(\pi_{\mathfrak{f}} \circ \varphi)(\mathbb{K} \oplus \mathbb{K})$ is either isomorphic to 0, $\mathbb{K}$, or $\mathbb{K} \oplus \mathbb{K}$. Hence, $(\pi_{\mathfrak{f}} \circ \varphi)(\mathbb{K} \oplus \mathbb{K}) = 0$ which implies that φ maps $\mathbb{K} \oplus \mathbb{K}$ to $\mathbb{K} \oplus \mathbb{K}$. Similarly,