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Optimal regularity of solutions to the obstacle problem for the fractional Laplacian with drift

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We prove existence, uniqueness and optimal regularity of solutions to the stationary obstacle problem defined by the fractional Laplacian operator with drift, in the subcritical regime. As in [4], we localize our problem by considering a suitable extension operator introduced in [2]. The structure of the extension equation is different from the one constructed in [4], in that the obstacle function has less regularity, and exhibits some singularities. To take into account the new features of the problem, we prove a new monotonicity formula, which we then use to establish the optimal regularity of solutions.

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Contents

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1. Introduction

We consider the linear operator defined by the fractional Laplacian with drift,

$$Lu(x) := (-\Delta)^s u(x) + b(x) \cdot \nabla u(x) + c(x)u(x), \quad \forall u \in C_c^2\big(\mathbb{R}^n\big), \tag{1.1}$$

where the coefficient functions $b : \mathbb{R}^n \to \mathbb{R}^n$ and $c : \mathbb{R}^n \to \mathbb{R}$ are assumed to be Hölder continuous. The action of the fractional Laplacian operator on functions $u \in C_c^2(\mathbb{R}^n)$ is given by the singular integral,

$$(-\Delta)^s u(x) = c_{n,s}\, \text{p.v.} \int_{\mathbb{R}^n} \frac{u(x) - u(y)}{|x-y|^{n+2s}}\, dy,$$

understood in the sense of the principal value. The constant $c_{n,s}$ is positive and depends only on the dimension $n \in \mathbb{N}$, and on the parameter $s \in (0,1)$. The range $(0,1)$ of the parameter s is particularly interesting because in this case the fractional Laplacian operator is the infinitesimal generator of the symmetric $2s$-stable process [1, Example 3.3.8].

The fractional Laplacian plays the same paradigmatic role in the theory of nonlocal operators that the Laplacian plays in the theory of local elliptic operators. For this reason, the regularity of solutions to equations defined by the fractional Laplacian and its gradient perturbation is intensely studied in the literature. In this article, we study the stationary obstacle problem defined by the fractional Laplacian operator with drift (1.1), in the subcritical regime, that is, the case when the parameter s belongs to the range $(1/2, 1)$. Given an obstacle function, $\varphi \in C^{3s}(\mathbb{R}^n) \cap C_0(\mathbb{R}^n)$, we prove existence, uniqueness and optimal regularity of solutions in Hölder spaces, $u \in C^{1+s}(\mathbb{R}^n)$, for the stationary obstacle problem,

$$\min\big\{(-\Delta)^s u(x) + b(x) \cdot \nabla u(x) + c(x)u(x), u(x) - \varphi(x)\big\} = 0, \quad \forall x \in \mathbb{R}^n. \tag{1.2}$$

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