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Minimal time for the null controllability of parabolic systems: The effect of the condensation index of complex sequences

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A B S T R A C T

Let $(\mathcal{A}, D(\mathcal{A}))$ be a diagonalizable generator of a C^0-semigroup of contractions on a complex Hilbert space $\mathbb{X}$, $\mathcal{B} \in \mathcal{L}(\mathbb{C}, Y)$, Y being some suitable extrapolation space of $\mathbb{X}$, and $u \in L^2(0, T; \mathbb{C})$. Under some assumptions on the sequence of eigenvalues $\Lambda = \{\lambda_k\}_{k\geq 1} \subset \mathbb{C}$ of $(\mathcal{A}, D(\mathcal{A}))$, we prove the existence of a *minimal* time $T_0 \in [0, \infty]$ depending on Bernstein's condensation index of Λ and on $\mathcal{B}$ such that $y' = \mathcal{A}y + \mathcal{B}u$ is null-controllable at any time $T > T_0$ and not null-controllable for $T < T_0$. As a consequence, we solve controllability problems of various systems of coupled parabolic equations. In particular, new results on the boundary controllability of one-dimensional parabolic systems are derived. These seem to be difficult to achieve using other classical tools.

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Condensation index of complex sequences

Contents

1. Introduction

The starting point of this paper is to deal with the controllability properties of non-scalar parabolic systems. Before describing the problem under consideration, let us recall some known results about the controllability properties of scalar parabolic systems. The null controllability problem for scalar parabolic systems has been first considered in the one-dimensional case. Let us consider the following null controllability problem: Given $y_0 \in H^{-1}(0,\pi)$, can we find a control $v \in L^2(0,T)$ such that the corresponding solution $y \in C([0,T]; H^{-1}(0,\pi))$ to

$$\begin{cases} \partial_t y - \partial_{xx} y = 0 & \text{in } Q := (0,\pi) \times (0,T), \\ y(0,\cdot) = v, \quad y(\pi,\cdot) = 0 & \text{on } (0,T), \\ y(\cdot,0) = y_0 & \text{in } (0,\pi), \end{cases} \tag{1.1}$$

satisfies

$$y(\cdot,T) = 0 \quad \text{in } (0,\pi)? \tag{1.2}$$

Using the moment method, H.O. Fattorini and D.L. Russell gave a positive answer to the previous controllability question (see [10] and [11]). Let us briefly recall the main ideas of this moment method.