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Periodically correlated multivariate second order random distribution fields and stationary cross correlatedness

Păstorel Gaşpar, Lavinia Sida *

Department of Mathematics and Computer Science, "Aurel Vlaicu" University of Arad, Arad, Romania

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In this paper we consider multivariate second order operator periodically correlated random distribution fields for which we complete and extend the results from [5].

1. Introduction

For the second order one time continuous parameter univariate continuous stochastic processes, the periodically correlatedness was first considered by E.G. Gladyshev in [7] and then developed in [8] and [13]. A. Makagon in [11] and [12] extended this study to

* Corresponding author.
E-mail addresses: pastorel.gaspar@uav.ro (P. Gaşpar), lavinia_sd@yahoo.com (L. Sida).

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univariate locally square integrable periodically correlated processes. A description of periodically correlated multivariate second order (m.s.o.) stochastic processes with two (or d) continuous time parameters in terms of stationarity, was given in [2] and [3], while in [5] the univariate second order random distribution fields in the case of continuous d time parameters are considered. In this paper we aim to describe the operator periodically correlated m.s.o. random distribution fields with d continuous time parameters in terms of a certain operator stationarily cross correlated family of m.s.o. random distribution fields.

Let us mention that another extension of periodically correlatedness for univariate stochastic processes was given in the recent paper [1], where the discrete time parameter and the continuous time parameter cases are treated in a unitary way, by considering such stochastic processes indexed on an arbitrary locally compact abelian group.

The rest of the first section is devoted to notations and basic definitions which are needed further on.

Given a probability space $(\Omega, \mathcal{A}, \wp)$ and an infinite dimensional complex separable Hilbert space E, we denote by $L_0^2(\wp, E)$ the normal Hilbert $\mathcal{B}(E)$-module of E-valued second order random variables of zero mean. When G is another complex separable Hilbert space, the space of all bounded linear operators from G to E will be denoted by $\mathcal{B}(G, E)$, while $\mathcal{C}^1(G, E)$ and $\mathcal{C}^2(G, E)$ mean the trace class, and the Hilbert–Schmidt class of operators from $\mathcal{B}(G, E)$, respectively. It is well known [10] that $L_0^2(\wp, E)$ is module isomorphic to $\mathcal{C}^2(L_0^2(\wp), E)$. Hence we shall use for both the notation $\mathcal{H}$. Recall that in $\mathcal{C}^2(G, E)$ the gramian is given by $[T, S] := TS^*$, $T, S \in \mathcal{C}^2(G, E)$ (see [10]). Let's add that when $\mathbb{R}^d$ is the d-dimensional euclidean space we denote by $m_d(\cdot)$ the Lebesgue measure on $\mathbb{R}^d$ and $\mathcal{D}_d$ denotes the locally convex space of compactly supported infinitely many times differentiable functions on $\mathbb{R}^d$ from distribution theory (see [14,4]).

Now, the multivariate second order (m.s.o.) random distribution field will be a mapping of the form

$$\mathcal{D}_d \ni \varphi \overset{U}{\longmapsto} U(\varphi) = U_\varphi \in \mathcal{H}, \tag{1}$$

which is linear and continuous, i.e. $U \in \mathcal{D}_d'(\mathcal{H})$. Its modular time domain $\mathcal{H}_U$ is the closure in $\mathcal{H}$ of the $\mathcal{B}(E)$-module $\mathcal{H}_U^0$, generated by the set $\{U_\varphi, \varphi \in \mathcal{D}\}$. The stationarity for m.s.o. random distribution fields is studied in [6]. In this paper we make currently use of the concepts and apply results from [10,5,6].

If V is another $\mathcal{H}$-valued s.o. random distribution field, then the $\mathcal{C}^1(E)$-valued sesquilinear function on $\mathcal{D}_d \times \mathcal{D}_d$ defined by

$$\Gamma_{U,V}(\varphi, \psi) = \big[U(\varphi), V(\psi)\big]_{\mathcal{H}}, \quad \varphi, \psi \in \mathcal{D}_d \tag{2}$$

is called the *operator cross correlation function of the fields U and V*. Let's observe that $\Gamma_{U,U}$ is the *operator correlation function of the field U* as was properly defined in [6].

Recall that the subspace $\mathcal{D}_d \otimes \mathcal{D}_d$, consisting of all finite sums of elementary tensors, i.e. of functions of the form