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Reconstruction and stability in acousto-optic imaging for absorption maps with bounded variation [☆]

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The aim of this paper is to propose for the first time a reconstruction scheme and a stability result for recovering from acousto-optic data absorption distributions with bounded variation. The paper extends earlier results in [3] and [5] on smooth absorption distributions. It opens a door for a mathematical and numerical framework for imaging, from internal data, parameter distributions with high contrast in biological tissues.

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1. Introduction

In the recent papers [3–5], an original mathematical and numerical framework for modeling biomedical imaging modalities based on mechanical perturbations of the medium was developed. The objective is to enhance the resolution and stability of tissue property imaging.

Many kinds of waves propagate in biological tissues over certain frequency ranges. Each one of them can be used to provide an image of a specific physical parameter. Low-frequency electromagnetic waves are sensitive to electrical conductivity, optical waves tell about optical absorption, ultrasonic waves reveal tissue's density, mechanical shear waves indicate how tissues respond to shear forces. However, single-wave imaging modalities are known to suffer from low specificity as well as intrinsic instabilities and low resolution; see [2] and [15]. These fundamental deficiencies are impossible to eliminate, unless additional a priori information is incorporated. Single-wave imaging modalities can only be used for anomaly detection. Expansion techniques for data analysis, which reduce the set of admissible solutions and the number of unknowns, allow robust and accurate reconstruction of the location and of some geometric features of the anomalies, even with moderately noisy data.

One promising way to overcome the inherent limits of single-wave imaging and provide a stable and quantitative reconstruction of a distribution of physical parameters is to combine different wave-imaging modalities; see again [2] and [15]. A variety of multi-wave imaging approaches are being introduced and studied. In such approaches, two or more types of physical waves are involved in order to overcome the individual deficiencies of each one of them and to combine their strengths. Because of the way the waves are combined, multi-wave imaging can produce a single image with the best contrast and resolution properties of the two waves.

Three different types of wave interaction can be exploited in multi-wave imaging [8]: (i) the interaction of one kind of wave with tissue can generate a second kind of wave; (ii) a low-frequency wave that carries information about the desired contrast can be locally modulated by a second wave that has better spatial resolution; (iii) a fast propagating wave can be used to acquire a spatio-temporal sequence of the propagation of a slower transient wave.

In [3] and [5], by mechanically perturbing the medium we proved both analytically and numerically the stability and resolution enhancement for reconstructing optical tissue parameters. We showed how the high contrast of optical tomography [6] can be coupled to the high resolution of the acoustic propagation in soft tissues. The use of mechanical perturbations of the medium modeled by acoustic equations in fluids enhances the resolution to the order of the front width of the acoustic wave, which propagates inside the object. It dramatically increases the low resolution of optical tomography [14].

This paper is a continuation and an extension of the work started in [3] and [5]. We keep here the same models for the diffusive light propagation [7] and for the acoustic perturbations. Our aim is to extend the reconstruction algorithm developed in [3] to