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Slow foliation of a slow–fast stochastic evolutionary system ☆

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This work is concerned with the dynamics of a slow–fast stochastic evolutionary system quantified with a scale parameter. An invariant foliation decomposes the state space into geometric regions of different dynamical regimes, and thus helps understand dynamics. A slow invariant foliation is established for this system. It is shown that the slow foliation converges to a critical foliation (i.e., the scale parameter is zero) in probability distribution, as the scale parameter tends to zero. The approximation of slow foliation is also constructed with error estimate in distribution. Furthermore, the geometric structure of the slow foliation is investigated: every fiber of the slow foliation parallels each other, with the slow manifold as a special fiber. In fact, when an arbitrarily chosen point of a fiber falls in the slow manifold, the fiber must be the slow manifold itself.

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Slow–fast stochastic evolutionary system
Geometric structure

1. Introduction

Random fluctuations may have delicate effects on dynamical evolution of complex systems [1,8,10,26]. The slow–fast stochastic evolutionary systems are appropriate mathematical models for various multi-scale systems under random influences.

We consider the following slow–fast stochastic evolutionary system

$$\frac{dx^\varepsilon}{dt} = Ax^\varepsilon + f\left(x^\varepsilon, y^\varepsilon\right) + \sigma_1 \dot{W}_1, \quad \text{in } H_s, \tag{1.1}$$

$$\frac{dy^\varepsilon}{dt} = \frac{1}{\varepsilon} By^\varepsilon + \frac{1}{\varepsilon} g\left(x^\varepsilon, y^\varepsilon\right) + \frac{\sigma_2}{\sqrt{\varepsilon}} \dot{W}_2, \quad \text{in } H_f, \tag{1.2}$$

where ε is a small positive parameter ($0 < \varepsilon \ll 1$). The Hilbert spaces H_s and H_f, linear operators A and B, nonlinearities f and g, and mutually independent Wiener processes W_1 and W_2 will be specified in the next section. The white noises $\dot{W}_1$ and $\dot{W}_2$ are the generalized time derivatives of W_1 and W_2, respectively. The positive constants σ_1 and σ_2 are the intensities of white noises. Since the small scale parameter ε is such that $\|\frac{dx}{dt}\|_{H_s} \ll \|\frac{dy}{dt}\|_{H_f}$, we usually say that x is the "slow" component and y is the "fast" component.

The main goal of this paper is to investigate state space decomposition for this system, by considering a slow invariant foliation, and examining its approximation and structure.

Invariant foliations and invariant manifolds play a significant role in the study of the qualitative dynamical behaviors, as they provide geometric structures to understand or reduce stochastic dynamics [4,5,7,11–13,18,20,21]. An invariant foliation is about quantifying certain sets (called fibers or leaves) in state space for a dynamical system. A fiber consists of all those points starting from which the dynamical orbits are exponentially approaching each other, in forward time ("stable foliation") or backward time ("unstable foliation"). These fibers are thus building blocks for understanding dynamics, as they carry dynamical information. Collectively they provide a decomposition of the state space.

For a system like (1.1)–(1.2), Schmalfuss and Schneider [22] studied the slow manifold in the finite dimensional case. Wang, Duan, and Roberts [25,24] further studied the slow manifold, and a relation with averaging as quantified via large deviations and approximations. In the infinite dimensional setting, Fu, Liu and Duan [15] investigated the slow manifold and its approximation. These research works are at the level of geometric and global invariant sets. In the context of analyzing individual sample solution paths, Freidlin [14] used large deviation theory to describe the dynamics, and Berglund and Gentz [3] showed that the sample solution paths are concentrated in a neighborhood of the critical manifold (also see [17]).