Journal of Functional Analysis 267 (2014) 2731–2786

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

The analytic torsion and its asymptotic behaviour for sequences of hyperbolic manifolds of finite volume

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A R T I C L E   I N F O

Article history:
Received 24 September 2013
Accepted 3 August 2014
Available online 2 September 2014
Communicated by P. Delorme

MSC:
primary 58J52
secondary 53C35

Keywords:
Analytic torsion
Locally symmetric manifolds

A B S T R A C T

In this paper we study the regularized analytic torsion of finite volume hyperbolic manifolds. We consider sequences of coverings X_i of a fixed hyperbolic orbifold X_0. Our main result is that for certain sequences of coverings and strongly acyclic flat bundles the analytic torsion divided by the index of the covering converges to the L^2-torsion. Our results apply to certain sequences of arithmetic groups, in particular to sequences of principal congruence subgroups of $\mathrm{SO}^0(d,1)(\mathbb{Z})$ and to sequences of principal congruence subgroups or Hecke subgroups of Bianchi groups.

1. Introduction

The aim of this paper is to extend the results of Bergeron and Venkatesh [4] on the asymptotic equality of analytic and L^2-torsion for strongly acyclic representations from the compact to the finite volume case.

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http://dx.doi.org/10.1016/j.jfa.2014.08.005

Therefore, we shall first recall the results of Bergeron and Venkatesh in the compact case. Let G be a semisimple Lie group of non-compact type. Let K be a maximal compact subgroup of G and let $\widetilde{X} = G/K$ be the associated Riemannian symmetric space endowed with a G-invariant metric. Let $\Gamma \subset G$ be a co-compact discrete subgroup. For simplicity we assume that Γ is torsion-free. Let $X := \Gamma\backslash\widetilde{X}$. Then X is a compact locally symmetric manifold of non-positive curvature. Let τ be an irreducible finite dimensional complex representation of G. Let $E_\tau \to X$ be the flat vector bundle associated to the restriction of τ to Γ. By [23], E_τ can be equipped with a canonical Hermitian fiber metric, called admissible, which is unique up to scaling. Let $\Delta_P(\tau)$ be the Laplace operator on E_τ-valued p-forms with respect to the metric on X and in E_τ. Let $\zeta_p(s;\tau)$ be the zeta function of $\Delta_p(\tau)$ (see [38]). Then the analytic torsion $T_X(\tau) \in \mathbb{R}^+$ is defined by

$$T_X(\tau) := \exp\left(\frac{1}{2}\sum_{p=1}^{d}(-1)^p p \frac{d}{ds}\zeta_p(s;\tau)\big|_{s=0}\right). \tag{1.1}$$

On the other hand, there is the L^2-torsion $T_X^{(2)}(\tau)$ (see [21,22]). Since the heat kernels on $\widetilde{X}$ are G-invariant, one has

$$\log T_X^{(2)}(\tau) = \operatorname{vol}(X) t_{\widetilde{X}}^{(2)}(\tau), \tag{1.2}$$

where $t_{\widetilde{X}}^{(2)}(\tau)$ is a constant that depends only on $\widetilde{X}$ and τ. It is an interesting problem to see if the L^2-torsion can be approximated by the torsion of finite coverings $X_i \to X$. This problem has been studied by Bergeron and Venkatesh [4] under a certain non-degeneracy condition on τ. Representations which satisfy this condition are called *strongly acyclic*. One of the main results of [4] is as follows. Let $X_i \to X$, $i \in \mathbb{N}$, be a sequence of finite coverings of X. Let τ be strongly acyclic. Let $\operatorname{inj}(X_i)$ denote the injectivity radius of X_i and assume that $\operatorname{inj}(X_i) \to \infty$ as $i \to \infty$. Then by [4, Theorem 4.5] one has

$$\lim_{i\to\infty} \frac{\log T_{X_i}(\tau)}{\operatorname{vol}(X_i)} = t_{\widetilde{X}}^{(2)}(\tau). \tag{1.3}$$

If $\operatorname{rk}_{\mathbb{C}}(G) - \operatorname{rk}_{\mathbb{C}}(K) = 1$, one can show that $t_{\widetilde{X}}^{(2)}(\tau) \neq 0$. Using the equality of analytic torsion and Reidemeister torsion [26], Bergeron and Venkatesh [4] used this result to study the growth of torsion in the cohomology of cocompact arithmetic groups. Furthermore, recently P. Scholze [35] has shown the existence of Galois representations associated with mod p cohomology of locally symmetric spaces for GL_n over a totally real or CM field. This makes it desirable to extend these results in various directions. Especially, one would like to extend (1.3) to the finite volume case. However, due to the presence of the continuous spectrum of the Laplace operators in the non-compact case, one encounters serious technical difficulties in attempting to generalize (1.3) to the finite volume case. In [32] J. Raimbault has dealt with finite volume hyperbolic 3-manifolds. In [33] he applied this to study the growth of torsion in the cohomology for certain sequences of