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A characterization of boundary conditions yielding maximal monotone operators

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A B S T R A C T

We provide a characterization for maximal monotone realizations for a certain class of (nonlinear) operators in terms of their corresponding boundary data spaces. The operators under consideration naturally arise in the study of evolutionary problems in mathematical physics. We apply our abstract characterization result to Port–Hamiltonian systems and a class of frictional boundary conditions in the theory of contact problems in visco-elasticity.

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1. Introduction

As it was shown in several articles [20–22,28,26,33–35,37,39] evolutionary problems in classical mathematical physics can often be written as a differential equation of the form

$$(\partial_0 \mathcal{M} + A)u = f,$$

where u is the unknown, f is a given source term, ∂_0 denotes the temporal derivative, $\mathcal{M}$ is a suitable bounded operator acting in space–time and A is a maximal monotone (possibly nonlinear) operator, which frequently is a suitable restriction of a block operator matrix of the form

$$\begin{pmatrix} 0 & D \\ G & 0 \end{pmatrix}, \tag{1}$$

where G and D are densely defined closed linear operators satisfying $-G^* \subseteq D$.

The aim of this article is to provide a characterization of all maximal monotone restrictions of (1). This characterization will be given in terms of the so-called boundary data spaces, introduced in [24,27], associated with the operators G and D. Moreover, we give a characterization of skew-selfadjoint restrictions of (1), which is a natural question arising for instance in the study of energy preserving evolutionary problems (see e.g. [25,24]).

The question of maximal monotone (or m-accretive) realizations of certain operators or relations was studied in various papers. For instance in 1959, Phillips [19] provides a characterization of m-accretive realizations of linear operators using indefinite metrics on Hilbert spaces on the one hand and the Neumann–Cayley transform on the other hand. Later on these results were generalized to linear relations in [8]. More recently, in [1] we find a characterization result for m-accretive extensions of linear relations in Hilbert spaces using the theory of Friedrichs- and Neumann-extensions of symmetric relations [5]. Another strategy to study extensions of operators or relations uses the theory of boundary triplets or, more general, boundary relations (see e.g. [6,2,7] and also [31] for so-called boundary systems). So, for instance in [10, Chapter 3] the question of m-accretive extensions of sectorial operators is addressed and a characterization is given in terms of boundary triplets. To the authors best knowledge all these strategies are